MAT1500-03/10 02F Quiz 8 Print Name (Last, First) ______________________ 10

last first

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. BOX final short answers. Always simplify expressions.

① Evaluate the following limits which would be necessary in graphing the function $f(x) = \dfrac{e^x - e}{x-1}$ : a) $\lim\limits_{x\to 1} f(x)$ b) $\lim\limits_{x\to -\infty} f(x)$ c) $\lim\limits_{x\to \infty} f(x)$ .

② Given $f(x) = x^{2/3}(x-6)^{1/3}$, $f'(x) = \dfrac{(x-4)}{x^{1/3}(x-6)^{2/3}}$, $f''(x) = \dfrac{-8}{x^{4/3}(x-6)^{5/3}}$,

a) draw the sign $(+-0)$ chart for $f'$ and the stick figure plot above it,

b) draw the sign chart for $f''$ labeled by curvature icons:

c) find the coordinates $(x,y)$ of the local max/mins (identify them as max or min) and other critical points, and of the points of inflection.

① a) $\lim\limits_{x\to 1} \dfrac{\overset{\to\, e^1 - e = 0}{e^x - e}}{\underset{\to\, 1-1=0}{x-1}} \sim \dfrac{0}{0} \xrightarrow[\text{rule}]{\text{l'Hospital's}} = \lim\limits_{x\to 1} \dfrac{\frac{d}{dx}(e^x - e)}{\frac{d}{dx}(x-1)} = \lim\limits_{x\to 1} \dfrac{e^x}{1} = e^1 = \boxed{e}$

b) $\lim\limits_{x\to -\infty} \dfrac{e^x - e \to e^{-\infty} - e = -e}{x-1 \to -\infty}$, $e^{-\infty} \to 0$, $\sim \dfrac{-e}{-\infty} = \boxed{0}$ determinant limit

c) $\lim\limits_{x\to \infty} \dfrac{e^x - e \to e^{\infty} = \infty}{x-1 \to \infty} \sim \dfrac{\infty}{\infty} \xrightarrow[\text{rule}]{\text{l'Hospital's}} = \lim\limits_{x\to \infty} \dfrac{\frac{d}{dx}(e^x - e)}{\frac{d}{dx}(x-1)} = \lim\limits_{x\to\infty} \dfrac{e^x}{1} = e^{\infty} = \boxed{\infty}$

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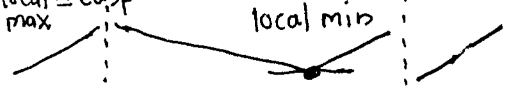
local max = cusp     local min     vertical tangent     f defined at all x

② a)



$f'$: $+$ $(\pm\infty)$ $-$ $-$ $0$ $+$ $(\pm\infty)$ $+$

0 4 6

b) $+$ $(\pm\infty)$ $+$ $(\pm\infty)$ $-$

0 4 6

c) crit points at $x = 0, 4, 6$ $\longrightarrow$ pts $\boxed{(0,0),\ (4, -2^{5/3}),\ (6,0) \text{ local max, local min}}$

$f(0) = 0^{2/3}(-6)^{1/3} = 0$

$f(4) = 4^{2/3}(-2)^{1/3} = -2^{4/3}\cdot 2^{1/3} = -2^{5/3} \approx 3.175$

$f(6) = 0$

$f''$ switches sign at $x = 6$ $\boxed{\text{pt } (6,0) = \text{pt of inflection}}$