MAT1500-03/10 02F Quiz 7 Print Name (Last, First) _____
last first

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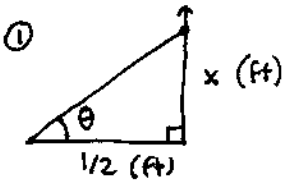
Show all work, including mental steps, in a clearly organized way that speaks for itself. User proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. BOX final short answers. Always simplify expressions.

① 

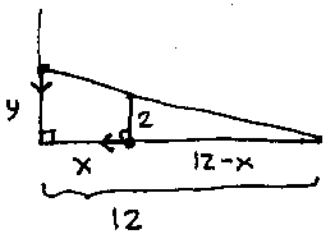
If $\theta = \arctan 2x$ and $\left.\frac{dx}{dt}\right|_{x=1/2} = 1/10$ ft/min, evaluate $\left.\frac{d\theta}{dt}\right|_{x=1/2}$. Recall $\frac{d}{dx}\arctan x = \frac{1}{1+x^2}$.

② A spotlight on the ground shines on a wall 12 m away. If a man 2 m tall walks from the spotlight toward the building at a speed of 1.6 m/s, how fast is the length of his shadow on the building decreasing when he is 4 m from the building? [Be sure to label the relevant variables in your diagram and state the rate of change you are given and the one you wish to calculate in terms of these variables.]

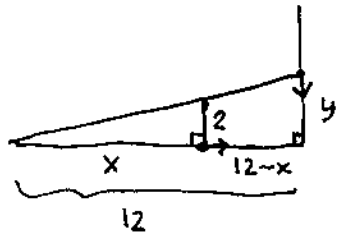
① $\theta = \arctan 2x$ $\quad \frac{d\theta}{dt} = \frac{d}{dt}(\arctan 2x) = \frac{1}{1+(2x)^2}\frac{d}{dt}(2x) = \frac{1}{1+4x^2}\left(2\frac{dx}{dt}\right)$

$\left.\frac{d\theta}{dt}\right|_{x=1/2} = \frac{1}{1+4(\frac{1}{2})^2}\left.2\frac{dx}{dt}\right|_{x=1/2} = \frac{2}{1+1}\left(\frac{1}{10}\right) = \boxed{\frac{1}{10}\ \frac{\text{rad}}{\text{min}}} \approx \frac{5.7^\circ}{\text{min}}$

②



or



$12-x=4 \rightarrow x=8$

$\frac{dx}{dt} = -1.6\ \frac{m}{s}$, $\quad -\left.\frac{dy}{dt}\right|_{x=4} = ?$

By similar triangles

$\frac{y}{12} = \frac{2}{12-x} \rightarrow y = \frac{24}{12-x}$

$\frac{dy}{dt} = \frac{d}{dt}24(12-x)^{-1} = 24(-1)(12-x)^{-2}\underbrace{\frac{d}{dt}(12-x)}_{-\frac{dx}{dt}}$

$= \frac{24}{(12-x)^2}\frac{dx}{dt}$

$-\left.\frac{dy}{dt}\right|_{x=4} = -\frac{24}{(12-4)^2}(-1.6) = \frac{(24)}{64}\left(\frac{16}{10}\right) = \frac{(3\cdot 8)(2\cdot 8)}{8\cdot 8\ 10}$

$= \frac{6}{10} = \frac{3}{5} = 0.6$

The length of his shadow is decreasing at $\boxed{0.6\ \text{m/s}}$.

$\frac{dx}{dt} = 1.6\ \frac{m}{s}$, $\quad -\left.\frac{dy}{dt}\right|_{x=8} = ?$

By similar triangles

$\frac{y}{12} = \frac{2}{x} \rightarrow y = \frac{24}{x}$

$\frac{dy}{dt} = \frac{d}{dt}(24x^{-1}) = 24(-1)x^{-2}\frac{dx}{dt}$

$= -\frac{24}{x^2}\frac{dx}{dt}$

$-\left.\frac{dy}{dt}\right|_{x=8} = \frac{24}{8^2}\frac{16}{10} = \frac{6}{10} = 0.6$