MAT1500-03/10 02F Quiz 6 Print Name (Last, First) _____
Last! first!

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Show all work, including mental steps, in a clearly organized way that speaks for itself. User proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. BOX final short answers. Always simplify expressions.

① Complete the following derivative formula list ("a" is a constant):

$\frac{d}{du}(u^a) =$ $\quad$ $\frac{d}{du}(\cos u) =$ $\quad$ $\frac{d}{du}(\tan u) = \sec^2 u$

$\frac{d}{du}(e^u) =$ $\quad$ $\frac{d}{du}(\sin u) =$ $\quad$ $\frac{d}{du}(\sec u) = \sec u \tan u$

② $y = \tan(e^{2x})$. Evaluate $\frac{dy}{dx}$ and $\frac{dy}{dx}\Big|_{x=1}$ exactly and to 4 decimal places.

③ pt. $(4,1)$, $\sqrt{x} + \sqrt{y} = x - y$ a) Evaluate $\frac{dy}{dx}$ and $\frac{dy}{dx}\Big|_{\substack{x=4\\y=1}}$.

b) Write an equation for the tangent line to this curve at $(4,1)$ and simplify it to standard slope intercept form.

① $\frac{d}{du}(u^a) = a u^{a-1}$ $\quad$ $\frac{d}{du}(\cos u) = -\sin u$

$\frac{d}{du}(e^u) = e^u$ $\quad$ $\frac{d}{du}(\sin u) = \cos u$

② $\frac{d}{dx}[y = \tan(e^{2x})] \rightarrow \frac{dy}{dx} = \frac{d}{dx}(\tan(e^{2x})) = \sec^2(e^{2x}) \underbrace{\frac{d}{dx}(e^{2x})}_{e^{2x}\underbrace{\frac{d}{dx}(2x)}_{2}}$

$= \boxed{2e^{2x}\sec^2 e^{2x}}$

$\frac{dy}{dx}\Big|_{x=1} = \boxed{2e^2 \sec^2 e^2 \approx 73.5144}$

③a) $\frac{d}{dx}(x^{1/2} + y^{1/2} = x - y) \rightarrow \frac{d}{dx}(x^{1/2} + y^{1/2}) = \frac{d}{dx}(x-y)$

$\frac{d}{dx}x^{1/2} + \frac{d}{dx}y^{1/2} = \frac{d}{dx}x - \frac{d}{dx}y$

$\frac{1}{2}x^{-1/2} + \frac{1}{2}y^{-1/2}\frac{dy}{dx} = 1 - \frac{dy}{dx}$

$\frac{1}{2}y^{-1/2}\frac{dy}{dx} + \frac{dy}{dx} = 1 - \frac{1}{2}x^{-1/2}$

$(1 + \frac{1}{2}y^{-1/2})\frac{dy}{dx} = 1 - \frac{1}{2}x^{-1/2}$

$\boxed{\frac{dy}{dx} = \frac{1 - \frac{1}{2}x^{-1/2}}{1 + \frac{1}{2}y^{-1/2}}}$ $= \frac{y^{1/2}}{x^{1/2}}\left(\frac{2x^{1/2}-1}{2y^{1/2}+1}\right)$ ↓ fraction algebra simpler?

b) pt $(4,1)$

slope $1/2$

$y - 1 = \frac{1}{2}(x-4)$

$y = 1 + \frac{1}{2}x - 2$

$\boxed{y = \frac{1}{2}x - 1}$

$\frac{dy}{dx}\Big|_{\substack{x=4\\y=1}} = \frac{1 - \frac{1}{2}\cdot\frac{1}{2}}{1 + \frac{1}{2}} = \frac{3/4}{3/2} = \frac{2}{4} = \boxed{\frac{1}{2}}$