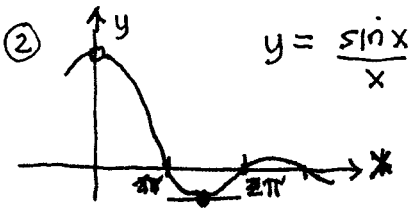
MAT1500-03/10 02F Quiz 5 Print Name (Last, First) _____ /10

LAST! FIRST!

Show all work, including mental steps, in a clearly organized way that speaks for itself. User proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. BOX final short answers. Always simplify expressions.

① $f(t) = te^t$ a) Calculate $f'(t)$ and factor your result.

b) Find the values of t for which $f'(t) = 0$.

② 

$y = \frac{\sin x}{x}$

a) Calculate dy/dx using the quotient rule.

b) Use your graphing calculator or Maple or Mathcad to numerically find the value of x where $dy/dx = 0$ for $0 < x < 2\pi$.

c) What value does y have there?

① a) $f(t) = te^t$ { not $\frac{dy}{dx}$, which is not the symbol for taking a derivative of an expression to its right, but the result of differentiating y by x.

$$f'(t) = \frac{d}{dt}(te^t) = \underbrace{\frac{d}{dt}(t)}_{1}\, e^t + t\underbrace{\frac{d}{dt}e^t}_{e^t} = e^t + te^t = \boxed{(1+t)e^t}$$

b) $0 = f'(t) = (1+t)\underbrace{e^t}_{\neq 0} \rightarrow 1+t = 0 \rightarrow \boxed{t = -1}$

② a) $y = \frac{\sin x}{x}$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\sin x}{x}\right) = \frac{x\overbrace{\frac{d}{dx}\sin x}^{\cos x} - \sin x\overbrace{\frac{d}{dx}x}^{1}}{x^2} = \boxed{\frac{x\cos x - \sin x}{x^2}}$$

b) $0 = \frac{dy}{dx} \rightarrow x\cos x - \sin x = 0$ ($\rightarrow$ or divide by $\cos x$: $\frac{x\cos x - \sin x}{\cos x} = 0$, $x - \tan x = 0$)

```
> fsolve( x*cos(x) - sin(x) = 0, x = Pi .. 2*Pi);
```

$x = \boxed{4.4934}$

c) $y = \frac{\sin x}{x} = \frac{\sin(4.4934)}{4.4934} = \boxed{-0.2172}$

The phrase "What value does y have there?" is ambiguous only if you are not thinking. This problem is about this graph y versus x shown in the figure. We found the x value where the tangent line is horizontal; clearly the next related information to find would be the y value of that point on the graph. Some had instead in their mind $y = \frac{d}{dx}\left(\frac{\sin x}{x}\right)$ and said, well $y = 0$ there. But why would I ask for that when setting "$y = \frac{d}{dx}\left(\frac{\sin x}{x}\right) = 0$" was how we found the x-value? We already knew that.