MAT1500-03/10 02F Quiz 3 Print Name (Last, First) ______________________________

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Show all work, including mental steps, in a clearly organized way that speaks for itself. User proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. BOX final short answers.

Evaluate the following limits exactly, showing the steps by which you reach your conclusion:

① $\lim\limits_{t\to 2} \frac{\frac{1}{t}-\frac{1}{2}}{t-2}$ ② $\lim\limits_{x\to -1} \frac{x^2-x-2}{x^2+3x+2}$ ③ $\lim\limits_{x\to -1^-} \frac{x^2+x+2}{x^2+3x+2}$ ④ $\lim\limits_{x\to -\infty} \frac{5x^3-x^2+2}{2x^3+x-3}$

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①

$\frac{1}{t}-\frac{1}{2} \to \frac{1}{2}-\frac{1}{2}=0$

$t-2 \to 2-2=0$

$\to \frac{0}{0}$ limit, need algebra:

$$\frac{\frac{1}{t}-\frac{1}{2}}{t-2} = \frac{\frac{2-t}{2t}}{t-2} = \frac{2-t}{2t(t-2)} = -\frac{(t-2)}{2t(t-2)} \overset{t\neq 2}{=} -\frac{1}{2t}$$

$$\lim_{t\to 2} \frac{\frac{1}{t}-\frac{1}{2}}{t-2} = \lim_{t\to 2} -\frac{1}{2t} = -\frac{1}{2\cdot 2} = \boxed{-\frac{1}{4}}$$

②

$x^2-x-2 \to 1+1-2=0$

$x^2+3x+2 \to 1-3+2=0$

$\to \frac{0}{0}$ limit, factors of $(x+1)$:

$$\frac{x^2-x-2}{x^2+3x+2} = \frac{\overset{1}{\cancel{(x+1)}}(x-2)}{\underset{1}{\cancel{(x+1)}}(x+2)} \overset{x\neq -1}{=} \frac{x-2}{x+2}$$

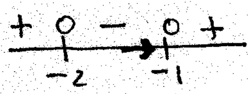
$$\lim_{x\to -1} \frac{x^2-x-2}{x^2+3x+2} = \lim_{x\to -1} \frac{x-2}{x+2} = \frac{(-1)-2}{(-1)+2} = -\frac{3}{1} = \boxed{-3}$$

③

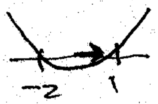
$x^2+x+2 \to 1-1+2=2$

$x^2+3x+2 \to 0$

$\to \frac{2}{0} \to \pm\infty$ check sign $\to$



sgn x^2+3x+2



$\doteq -1$ as approach -1 from the left

numerator $\to 2 > 0$

overall sign negative.

$$\lim_{x\to -1^-} \frac{x^2+x+2}{x^2+3x+2} = \lim_{x\to -1^-} \frac{2 > 0}{\underbrace{(x+2)}_{\to 1 > 0}\underbrace{(x+1)}_{<0 \text{ as } x\to -1^-}} = \boxed{-\infty}$$

or:

④

$$\lim_{x\to -\infty} \frac{5x^3-x^2+2 \;/x^3}{2x^3+x-3 \;/x^3} = \lim_{x\to -\infty} \frac{5-\cancel{\frac{1}{x}}^{\to 0}+\cancel{\frac{2}{x^2}}^{\to 0}}{2+\cancel{\frac{1}{x^2}}_{\to 0}-\cancel{\frac{3}{x^3}}_{\to 0}} = \boxed{\frac{5}{2}}$$

$$\left(\text{or} = \lim_{x\to -\infty} \frac{5\cancel{x^3}^{1}}{2\cancel{x^3}_{1}} = \lim_{x\to -\infty} \frac{5}{2} = \frac{5}{2}\right)$$

highest powers in sum