MAT1500-03/10 02F Quiz 2 Print Name (Last, First) _____ 10

= (this helps for alphabetization of papers)

Show all work, including mental steps, in a clearly organized way that speaks for itself. User proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. BOX final short answers.

The half-life of palladium-100, ^{100}Pd, is four days. (So half of any given quantity of ^{100}Pd will disintegrate in four days.) The initial mass of a sample is 1 g.

a) Find the mass that remains after 16 days.

b) Find the mass $m(t)$ that remains after t days.

c) Find the mass that remains after 29 days.

d) Is there more or less than 1% of the initial mass left after 29 days?

e) Graph the population function (graphing calculator or MAPLE) and estimate the time for the mass to decay to 1% of its initial value. (Explain what you do in words/sketch on this sheet.) (significant digits?)

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① a) $\frac{16}{4} = 4$ half lives so it decreases by 4 factors of 2 $\Big\}$ $1\text{ g}\left(2^{-4}\right) = \frac{1}{16}\text{ g} = \boxed{.0625\text{ g}}$

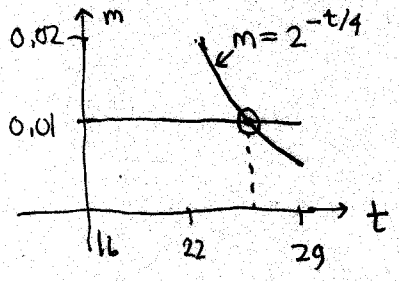
b) $\boxed{m(t) = \underbrace{1}_{m_0} \cdot 2^{-t/4}} = \boxed{2^{-t/4}}$ (g) ← half life

Note: the function $m(t)$ is the number of g of mass so the dimension is understood, so I put it in parentheses

c) $m(29) = 2^{-29/4} \approx \boxed{.00657\text{ (g)}}$

d) Since $m_0 = 1$, this is about 0.66 percent, which is $\boxed{\text{less than 1 percent}}$, namely .01 g.

e) We want to see where the function crosses the line $m = .01$ and we know it occurs between 16 and 29. One can then narrow the horizontal window to the interval $[26, 28]$ and see that $.01$ and $2^{-t/4}$ cross at roughly 26.5 or 26.6. Solving $2^{-t/4} = .01$ with technology gives a better precision of 26.6 to 3 significant digits.



Note: the next section (not allowed on this quiz) shows us how to solve the equation $\ln\left[2^{-t/4} = .01\right]$

$$-\tfrac{t}{4}\ln 2 = \ln .01 \qquad (10^{-2})$$

$$t = -4\,\frac{\ln .01}{\ln 2} = 8\,\frac{\ln 10}{\ln 2} \approx 26.58$$