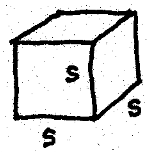
MAT1500-03/10 02F Quiz 1 Print Name (Last, First) ____________________ $\frac{}{10}$

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. BOX final short answers.

① 

Express the surface area of a cube as a function of its volume. (Choose appropriate variable names. Simplify your expression for the function.)

② Find the domain and range of the function defined by $f(x) = \sqrt{4-3x^2}$.

① $V = s^3$ (product of side lengths)

$S = 6s^2$ (six sides of area s^2, each a square)

goal: $S = S(V)$

→ solve for s: $V = s^3 \rightarrow V^{1/3} = (s^3)^{1/3} = s$

→ replace s: $S = 6\,(V^{1/3})^2 \underset{\text{simplify}}{=} 6V^{2/3}$ so $\boxed{S = 6V^{2/3}}$

note: math is case sensitive so lowercase and uppercase letters are different symbols — be consistent. In this case using A for surface area would avoid misinterpretation of S and s.

② $f(x) = \sqrt{4-3x^2} = (\underbrace{4-3x^2}_{})^{1/2}$

domain? ≥ 0 for real result (even roots of negative #s are complex)

$4-3x^2 \geq 0$

$4 \geq 3x^2$

$\frac{4}{3} \geq x^2$

simplify

$\frac{2}{\sqrt{3}} = \sqrt{\frac{4}{3}} \geq \sqrt{x^2} = |x|$

$\frac{2}{\sqrt{3}} \geq |x|$

so $\boxed{-\frac{2}{\sqrt{3}} \leq x \leq \frac{2}{\sqrt{3}}}$

or $\boxed{x \in \left[-\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}\right]}$

domain ↑

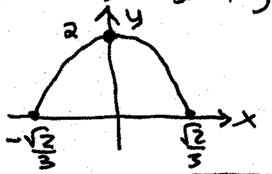
note: always retain "exact" numbers and use decimal approximations only when appropriate (for interpretation)

range?

largest value of $\sqrt{4-3x^2}$ occurs when $3x^2 = 0$ or $x = 0$ so that it reduces to $\sqrt{4} = 2$, i.e. $f(0) = \sqrt{4} = 2$ is the largest value.

the smallest value is obviously 0 which occurs at the endpoints of the domain where the input of the square root goes to 0

and it seems clear you can get every value in between:



so range: $\boxed{y \in [0, 2]}$

or $\boxed{0 \leq y \leq 2}$