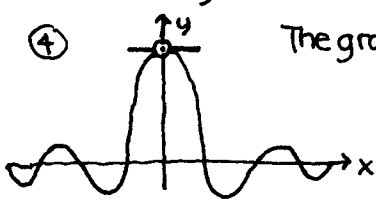


Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. BOX final short answers. [See long instructions on reverse].

① $y = (3x-2)^4 (2x-3)^3$. Find the (exact) values of x which are critical numbers for the graph of y .

② If $s(t) = 2 \sin^2 t - \cos(4t)$, what is $s''(\pi/2)$? (exactly)

③ Find the (slope-intercept form of the) equation of the tangent line to the curve $\sqrt{x} + \sqrt{y} = 3$ at the point $(4, 1)$.



④ The graph of $f(x) = \frac{\sin x}{x}$ has limiting value 1 and limiting slope 0 as x approaches 0. Verify this by symbolically evaluating

a) $\lim_{x \rightarrow 0} f(x)$ b) $\lim_{x \rightarrow 0} f'(x)$.

⑤^{a)} Find the local and global extreme values of the function $f(x) = x - \sqrt{x}$ on the interval $0 \leq x \leq 4$. [Remember to give the values of f , not just the x values and identify them as local and global max and mins and justify your claims.]

b) For the same function evaluate the linear approximation $L(x)$ at $x=4$ and use it to approximate $f(3.9)$.

c) Based on the value of $f''(4)$, is your approximation too high or too low? Explain.

⑥ In appropriate units, the speed of a wave of length L in deep water is $v = \sqrt{L + L^{-1}}$. Using the differential approximation, what is the approximate percentage change in v when $L=2$ and L increases by 10%? Does v increase or decrease?

[Recall the percentage change is the change of a quantity divided by its starting value and then multiplied by 100.]

⑦ A balloon is rising at a constant rate of 5 ft/s. A girl is cycling along a straight level road at a speed of 15 ft/s. When the girl passes under the balloon it is 45 ft above her. How fast is the distance between the girl and the balloon increasing 3 seconds later? [Be sure to make a diagram illustrating the situation at some time after the crossing point and indicating your choice of variables.]

⑧ A poster is to have an area of 180 in^2 with 1 in margins on the bottom and sides and a 2 in margin on the top. What dimensions will give the largest printed area?

[Give a diagram showing your choice of variables. Identify the allowed interval of values of the independent variable you use. Confirm the local max you find with the first derivative test.]

⑨ $f(x) = \arctan(3x) - \arctan(x)$ a) Get a good technology window view of the graph of f and check for asymptotes. Confirm your guess by numerically testing values. Report results.

b) Recalling that $\frac{d}{dx} \arctan x = \frac{1}{1+x^2}$, evaluate, simplify and factor $f'(x)$ and find its zeros exactly. c) Give the sign chart number line for f' and sketch the stick figure graph above it.

d) The second derivative is $f''(x) = \frac{4x(-13-18x^2+27x^4)}{(1+9x^2)^2(1+x^2)^2}$. Using technology, find the zeros of f'' (to 4 dec places if possible) and then give the sign chart for f'' .

e) List the coordinates (x,y) of all critical points of f (identify as local max/min/neither) and of all points of inflection (to 4 dec places if possible).

f) Give a hand graph of f reflecting what you have found, labeling all key points.

Math Exam Rules

READ THESE INSTRUCTIONS CAREFULLY

This test is not about just getting "the right answer", but also presenting and communicating well the process which leads to the results requested in each part of every problem, as well as your understanding of the course content and its vocabulary. [This is good practice for learning how to communicate technical results to other people in a workplace environment.] No results here may be justified using technology — a reasoned explanation supported by mathematical facts is always required and cannot be substituted by a technology result (except for antiderivatives which cannot be obtained directly or by using a u -substitution). However, you are encouraged to use MAPLE to check every result you derive by hand. [For a take home exam, no collaboration is allowed but you may consult your textbook, your notes and my handouts.] Come talk to me if you get stuck on any problem.

Show **all** work and answers, including indications of mental steps, on the lined paper provided. If you copy over work, be sure you include everything. Put your name on each sheet and clearly label continuations of problems from one sheet to another. Label and SEPARATE clearly each part of each problem and BOX each short final response requested (and nothing else). Cross out abandoned work not to be considered.

Use proper mathematical notation: "symbol" = "expression representing symbol" = "...", introducing symbols if necessary. Don't misuse equal signs, and don't write down unidentified expressions, but do link expressions which are equal with equal signs, while using arrows or colons to link expressions which are not equal but are related by some step. Give EXACT ANSWERS, not decimal approximations, unless the context warrants it, but first give the exact result in any case. Always simplify results. **Math is case sensitive: always be consistent with your upper and lower case letters used as symbols.**

When you have completed the exam, please read and sign the dr bob integrity pledge and attach it to your answer sheets (staple take home test) as a cover page, first side face up:

"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated."

Signature:

Date:

Word Problem Alternatives

If you get stuck in translating word problems 7 and/or 8 into corresponding "math" problems for whatever reason, you can instead do one of the following "math" problems for reduced credit.

(7b) A point moves along the curve $\sqrt{x} + \sqrt{y} = 3$ of problem 3 in a plane where the units are centimeters. If y is decreasing at 2 cm/sec when $x = 4$, how fast is x changing? (inc or dec?).

(8b) Find two positive integers such that the sum of the first number and four times the second number is 1000 and the product is as large as possible.