MAT1500-02/10 02F Test 3 takehome Answers

① a) $f(x) = \dfrac{e^x}{1+x^2}$, $f'(x) = \dfrac{(1+x^2)\frac{d}{dx}e^x - e^x\frac{d}{dx}(1+x^2)}{(1+x^2)^2}$

$= \dfrac{(1+x^2)e^x - e^x(2x)}{(1+x^2)^2} = \dfrac{e^x(1+x^2-2x)}{(1+x^2)^2}$

$= \boxed{\dfrac{e^x(1-x)^2}{(1+x^2)^2}}$ (fully factored)

b) $f'(x) = 0 \to (1-x)^2 = 0 \to \boxed{x=1}$

② a) $y = xe^{-2x}$, $\dfrac{dy}{dx} = \dfrac{d}{dx}(x)\,e^{-2x} + x\dfrac{d}{dx}e^{-2x}$

$= 1\cdot e^{-2x} + xe^{-2x}(-2) = (1-2x)e^{-2x}$

$\dfrac{d^2y}{dx^2} = \dfrac{d}{dx}(1-2x)e^{-2x} + (1-2x)\dfrac{d}{dx}e^{-2x}$

$= -2e^{-2x} + (1-2x)e^{-2x}(-2) = (4x-4)\,e^{-2x}$

$= \boxed{4(x-1)e^{-2x}} \underset{b)}{=} 0 \to \boxed{x=1}$

③ $y = \sqrt{\dfrac{x^2+1}{x^2-1}} = \left(\dfrac{x^2+1}{x^2-1}\right)^{1/2}$

$\ln y = \ln\left(\left(\dfrac{x^2+1}{x^2-1}\right)^{1/2}\right) = \dfrac{1}{2}\ln\left(\dfrac{x^2+1}{x^2-1}\right) =$

$= \dfrac{1}{2}\left(\ln(x^2+1) - \ln(x^2-1)\right)$

$\dfrac{d}{dx}(\ln y) = \dfrac{d}{dx}\dfrac{1}{2}\left(\ln(x^2+1) - \ln(x^2-1)\right)$

$\dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{1}{2}\left(\dfrac{d}{dx}\ln(x^2+1) - \dfrac{d}{dx}\ln(x^2-1)\right)$

$= \dfrac{1}{2}\left(\dfrac{2x}{x^2+1} - \dfrac{2x}{x^2-1}\right) = x\left(\dfrac{1}{x^2+1} - \dfrac{1}{x^2-1}\right)$

$= x\left(\dfrac{x^2-1-(x^2+1)}{(x^2+1)(x^2-1)}\right) = \dfrac{-2x}{(x^2+1)(x^2-1)}$

$\dfrac{dy}{dx} = \dfrac{(x^2+1)^{1/2}}{(x^2-1)^{1/2}}\,\dfrac{(-2x)}{(x^2+1)(x^2-1)} = \boxed{\dfrac{-2x}{(x^2+1)^{1/2}(x^2-1)^{3/2}}}$

④ a) $\dfrac{d}{dx}\left[xe^y = y-1\right]$

$\dfrac{d}{dx}(x)\,e^y + x\dfrac{d}{dx}e^y = \dfrac{d}{dx}(y) - \dfrac{d}{dx}(1)$

$e^y + xe^y\dfrac{dy}{dx} = \dfrac{dy}{dx} - 0$

$xe^y\dfrac{dy}{dx} - \dfrac{dy}{dx} = -e^y$

$(xe^y - 1)\dfrac{dy}{dx} = -e^y \to \boxed{\dfrac{dy}{dx} = \dfrac{e^y}{1-xe^y}}$

b) $\left.\dfrac{dy}{dx}\right|_{x=-1,\,y=0} = \dfrac{e^0}{1-(-1)e^0} = \dfrac{1}{1+1} = \dfrac{1}{2}$

$y - 0 = \frac{1}{2}(x-(-1)) \to \boxed{y = \frac{1}{2}(x+1)}$

$\boxed{y = \frac{1}{2}x + \frac{1}{2}}$

⑤ a) $s = 2\cos t + 3\sin t$

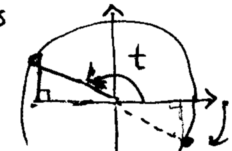
$\boxed{v = \dfrac{ds}{dt} = -2\sin t + 3\cos t}$

b) see reverse

⑤ c) $s = 2\cos t + 3\sin t = 0 \to 3\sin t = -2\cos t$

$\to \dfrac{\sin t}{\cos t} = -\dfrac{2}{3} \to \tan t = -\dfrac{2}{3} \to t = \ldots ?$

we want the first positive value of t that satisfies this

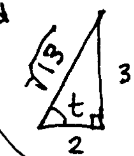


so $t = \boxed{\pi - \arctan\frac{2}{3} \approx 2.5536 \text{ (sec)}}$ agrees with plot

d) we need to evaluate s at its first local max, which occurs when $s' = 0$:

$0 = -2\sin t + 3\cos t \to \tan t = \dfrac{\sin t}{\cos t} = \dfrac{3}{2}$

$t = \arctan\frac{3}{2} \approx 0.9880 \to$



$\cos t = \dfrac{2}{\sqrt{13}}$

$\sin t = \dfrac{3}{\sqrt{13}}$

$s(t) = 2\left(\dfrac{2}{\sqrt{13}}\right) + 3\left(\dfrac{3}{\sqrt{13}}\right)$

$= \dfrac{4+9}{\sqrt{13}} = \dfrac{13}{\sqrt{13}} = \sqrt{13} \approx 3.606$ (cm) agrees with plot

e) We need to find the local min or max of V

$V = -2\sin t + 3\cos t$

$V' = -2\cos t - 3\sin t = 0 \to$

$\to \tan t = \dfrac{\sin t}{\cos t} = -\dfrac{2}{3}$ same as condition for $s = 0$

The speed is the greatest when the mass is passing through the equilibrium position.

also agrees with the plot. (2 per cycle)

add π to c) to get second time

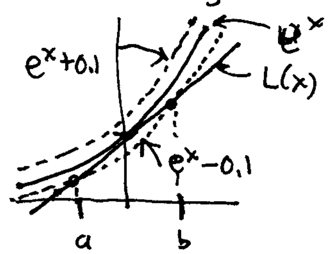
$2\pi - \arctan\frac{2}{3} \approx 5.6952$ (sec)

⑥ $f(x) = e^x \quad f'(x) = e^x$

$f(0) = e^0 = 1, \quad f'(0) = 1$

pt $(0,1)$, slope 1: $y - 1 = 1(x-0)$

$\to y = 1 + x \to \boxed{L(x) = 1+x}$



clearly the tan line crosses the lower curve on both sides so we must solve

$e^x - 0.1 = 1 + x$

$e^x - x - 1.1 = 0$

$\approx -0.48 \quad \approx 0.41$ (click on graph, graphing calc.)

$\approx -0.4832 \quad \approx 0.4162$ (fsolve plus roundoff)

⑦ $F = kR^4 \quad \dfrac{dF}{dR} = k(4R^3) \to dF = 4kR^3\,dR$

$\dfrac{dF}{F} = \dfrac{4kR^3\,dR}{kR^4} = 4\,\dfrac{dR}{R}$ ✓

approximate relative change in F in differential approximation | 4 times | relative change in R

$5\% \to .05 = \dfrac{dR}{R}$

$\to \dfrac{dF}{F} = 4(.05) = .20$

Blood flow increases by about $\boxed{20\%}$

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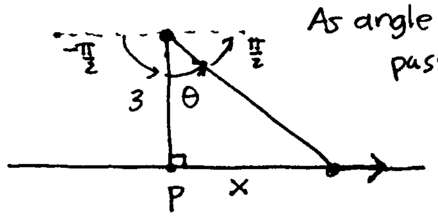
⑧ $y = (1+x^3)^{1/2}, \quad \left.\frac{dy}{dt}\right|_{x=2,\,y=3} = 4\left(\frac{cm}{s}\right), \quad \left.\frac{dx}{dt}\right|_{x=2,\,y=3} = ?$

$$\frac{dy}{dt} = \frac{d}{dt}(1+x^3)^{1/2} = \frac{1}{2}(1+x^3)^{-1/2}\underbrace{\frac{d}{dt}(1+x^3)}_{0+3x^2\frac{dx}{dt}} = \frac{3x^2}{2(1+x^3)^{1/2}}\frac{dx}{dt} \rightarrow \frac{dx}{dt} = \frac{2(1+x^3)^{1/2}}{3x^2}\frac{dy}{dt}$$

$$\left.\frac{dx}{dt}\right|_{x=2,\,y=3} = \frac{2(1+2^3)^{1/2}}{3\cdot 2^2}(4) = \frac{2\,9^{1/2}}{3} = \frac{2\cdot 3}{3} = \boxed{2\ \frac{cm}{s}} \quad \text{(increasing)}$$

⑨ 

As angle of spotlight direction increases, x increases (if it has already passed point P). Goal: $\left.\frac{dx}{dt}\right|_{x=1} = ?$

$$\tan\theta = \frac{x}{3} \rightarrow \theta = \arctan\left(\frac{x}{3}\right)$$

$$\frac{d\theta}{dt} = \frac{d}{dt}\left(\arctan\left(\frac{x}{3}\right)\right) = \frac{1}{1+\left(\frac{x}{3}\right)^2}\frac{d}{dt}\left(\frac{x}{3}\right) = \frac{1}{1+x^2/9}\left(\frac{1}{3}\frac{dx}{dt}\right) \rightarrow \frac{dx}{dt} = 3\left(1+\frac{x^2}{9}\right)\frac{d\theta}{dt}$$

4 revolutions per minute = 8π rad/min = $\frac{d\theta}{dt}$
(2π rad per revolution) units of length = km

$$\left.\frac{dx}{dt}\right|_{x=1} = 3\left(1+\frac{1}{9}\right)\cdot 8\pi = 3\left(\frac{10}{9}\right)8\pi = \boxed{\frac{80\pi}{3}\ \frac{km}{min}} \approx 83.776 \approx \boxed{84\ \frac{km}{min}}$$

of course when $-\frac{\pi}{2} < \theta < 0$, x is decreasing and its derivative is minus this value, but the speed of the point is positive and has the same value.

Stewart problem key:

① 3.R.4
② 3.7.14
③ 3.8.38 (simplified)
④ 3.R.34
⑤ 3.4.32
⑥ 3.11.14
⑦ 3.11.44
⑧ 3.10.4
⑨ 3.10.32

⑤ b

```
> s := t->2*cos(t)+3*sin(t);
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$s := t \rightarrow 2\cos(t) + 3\sin(t)$

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> plot([s(t),D(s)(t)],t=0..2*Pi,color=[red,blue]);
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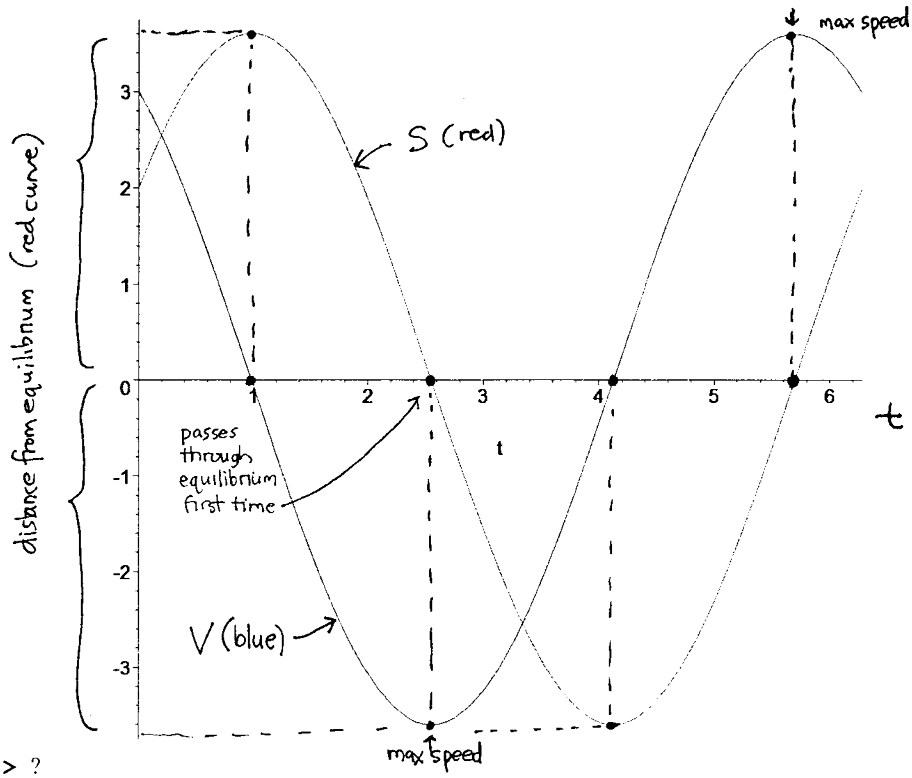


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> ?
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