MAT1500-03/10 02F TEST 2 ANSWERS

① a) $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

b) $f(x) = 2x^2-3x+1$

$$f'(x) = \lim_{h\to 0} \frac{2(x+h)^2-3(x+h)+1-[2x^2-3x+1]}{h}$$

$$= \lim_{h\to 0} \frac{2(\cancel{x^2}+2xh+h^2)-\cancel{3x}-3h+\cancel{1}-\cancel{2x^2}+\cancel{3x}-\cancel{1}}{h}$$

$$= \lim_{h\to 0} \frac{4xh+2h^2-3h}{h} = \lim_{h\to 0} \frac{\cancel{h}(4x+2h-3)}{\cancel{h}}$$

$$= \lim_{h\to 0} 4x+2h-3 = \boxed{4x-3}$$

c) $f'(-1) = 4(-1)-3 = -7$

$f(-1) = 2(-1)^2-3(-1)+1 = 2+3+1 = 6$

$y-6 = -7(x-(-1))$

$y = 6-7(x+1) = 6-7x-7 = -1-7x$

$\boxed{y=-1-7x}$

② a) $\lim_{x\to a} g(x) = g(a)$

b) $g(x) = \begin{cases} 2x+1 & x<0 \\ 2x-x^2 & 0\le x\le 2 \\ 2-x & x>2 \end{cases}$

g is a piecewise continuous function — just need to check continuity at 2 join points.

$\lim_{x\to 0^-} g(x) = \lim_{x\to 0^-} 2x+1 = 1$ $\neq$

$\lim_{x\to 0^+} g(x) = \lim_{x\to 0^+} 2x-x^2 = 0 = g(0)$

$\lim_{x\to 0} g(x)$ D.N.E. (jump discontinuity)

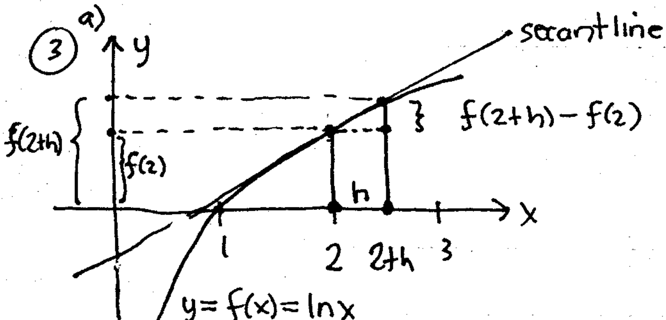
$\lim_{x\to 2^-} g(x) = \lim_{x\to 2^-} 2x-x^2 = 2\cdot 2-2^2 = 0$

$\lim_{x\to 2^+} g(x) = \lim_{x\to 2^+} 2-x = 2-2 = 0$

$\therefore \lim_{x\to 2} g(x) = 0 = g(2)$ continuous.

$\boxed{\text{summary: continuous everywhere except at } x=0}$

③ a)



b) $f'(2) = \lim_{h\to 0} \frac{f(2+h)-f(2)}{h} = \boxed{\lim_{h\to 0} \frac{\ln(2+h)-\ln 2}{h}}$

③ b) $= \lim_{h\to 0} \frac{\ln\left(\frac{2+h}{2}\right)}{h} = \lim_{h\to 0} \frac{\ln(1+\frac{h}{2})}{h}$

convenient for easier evaluation but not necessary

h:	0.1	0.01	0.001	0.0001
$\frac{\ln(1+\frac{h}{2})}{h}$:	0.4879,	0.4987,	0.49987,	0.49999

h:	−0.1	−0.01	−0.001	−0.0001
$\frac{\ln(1+h/2)}{h}$:	0.5129,	0.503,	0.5001,	0.50001

looks like converging to $0.5 = \boxed{1/2 = f'(2)}$

c) $f(2) = \ln 2$

$y-\ln 2 = \frac{1}{2}(x-2) \to y = \ln 2 + \frac{1}{2}x - 1$

$\boxed{y = (\ln 2 - 1) + \frac{1}{2}x}$

d) $\boxed{y = -0.3069 + 0.5000\,x}$

y intercept <0, hits y-axis $\boxed{\text{below origin}}$

④ $f(x) = 2x^3+x^2+2 = 0$ continuous

a) $f(0) = 2$

$f(-1) = -2+1+2 = 1$

same sign, IVT only guarantees f takes all values between 1 & 2.

$\boxed{\text{no conclusion possible.}}$

b) $f(-1) = 1$

$f(-2) = -16+4+2 = -10$

opp sign, f must assume value 0 on interval $[-2,-1]$

$\boxed{\text{so has a root in this interval.}}$

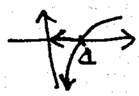
⑤ a) $\lim_{t\to 4} \frac{t-4}{\underbrace{t^2-3t-4}_{\text{must have factor of } t-4}} = \lim_{t\to 4} \frac{\cancel{t-4}}{(\cancel{t-4})(t+1)} = \lim_{t\to 4} \frac{1}{t+1} = \boxed{\frac{1}{5}}$

$t-4 \to 0$, $t^2-3t-4 \to 16-12-4 = 0$

b) $\lim_{t\to -1^-} \frac{t-4}{t^2-3t-4} = \lim_{t\to -1^-} \frac{(\cancel{t-4})}{(\cancel{t-4})(t+1)} = \lim_{t\to -1^-} \frac{1}{(t+1)} = \boxed{-\infty}$

$t-4 \to -5$, $t^2-3t-4 \to 0$, $(t+1) < 0$

c) $\lim_{x\to 10^-} \ln(\underbrace{100-x^2}_{\to 0^+}) = \boxed{-\infty}$

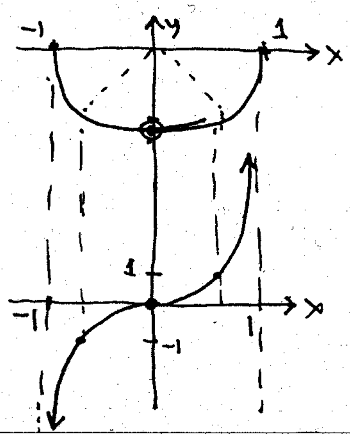


ln goes to $-\infty$ at 0.

d) $\lim_{x\to\infty} \frac{x-4}{x^2-3x-4} = \lim_{x\to\infty} \frac{x}{x^2} = \lim_{x\to\infty} \frac{1}{x} = \boxed{0}$

$\left(\text{LL} \lim_{x\to\infty} \frac{x-4\ /x^2}{x^2-3x-4\ /x^2} = \lim_{x\to\infty} \frac{\frac{1}{x}-\frac{4}{x^2}}{1-\frac{3}{x}-\frac{4}{x^2}} = \frac{0}{1} = \boxed{0} \right)$

⑥



slope 0 at origin (hor tan line), positive for (0,1) goes to ∞ at x=1 (vert tan line).

sign reversed on left half.

b) domain f': $(-1,1)$

range f': $(-\infty,\infty)$

(infinite values at endpoints, takes all values in between)