MAT1500-03/10 02F Test 2 Print Name (Last, First) ____________________

Show all work, including mental steps, in a clearly organized way that speaks for itself. User proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. BOX final short answers. [See long instructions on reverse].

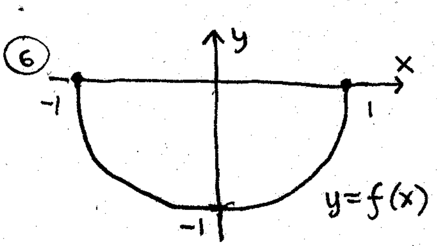
1. a) State the (most convenient form of the) limit definition of the derivative of a function $f$.
   b) Use the limit definition to evaluate $f'(x)$ for the function $f(x) = 2x^2 - 3x + 1$.
   c) Write the equation of the tangent line to the graph of $f$ at $x = -1$, simplifying your result to the slope-intercept form $y = mx + b$.

2. a) State the condition (expressed as a mathematical equation) that a function $g$ is continuous at $x = a$.
   b) If $g(x) = \begin{cases} 2x+1, & x < 0 \\ 2x - x^2, & 0 \le x \le 2 \\ 2 - x, & x > 2 \end{cases}$, where is this function continuous? Support any claims you make by referring to the condition in part a).

3. a) Make a rough (caricature) graph of the function $f(x) = \ln x$ in the x-y plane, for $0 \le x \le 3$. On this graph mark and label line segments that represent $f(2)$, $f(2+h)$, $f(2+h) - f(2)$, and $h$ (assuming $h > 0$). Draw in the line which has slope $\frac{f(2+h) - f(2)}{h}$ and passes through the graph of $f$ at $x = 2$.
   b) Write down the limit definition for $f'(2)$ for this function $f$ and estimate its value by evaluating the expression inside the limit first for $h = [0.1, 0.01, 0.001, 0.0001]$ and then for $h = [-0.1, -0.01, -0.001, -0.0001]$. Guess from these results what the exact value of $f'(2)$ should be.
   c) Use your final result from part b) to write an exact equation for the tangent line to the graph of $f$ at $x = 2$ and simplify your result to the slope-intercept form.
   d) Approximate your final result to part c) to 4-decimal place accuracy. Does this tangent line pass through the y-axis above or below the origin? Why?

4. Based on the intermediate value theorem alone, can one conclude that $2x^3 + x^2 + 2 = 0$ has a root (solution) in the interval: a) $[-1, 0]$? b) $[-2, -1]$? For each part, explain why or why not.

5. Evaluate the following limits exactly (ie not based on guesses from calculator values). Support your results by showing your reasoning (use words if you do not easily see how to document your work otherwise).
   a) $\lim\limits_{t \to 4} \frac{t-4}{t^2 - 3t - 4}$ b) $\lim\limits_{t \to -1^-} \frac{t-4}{t^2 - 3t - 4}$ c) $\lim\limits_{x \to 10^-} \ln(100 - x^2)$ d) $\lim\limits_{x \to \infty} \frac{x-4}{x^2 - 3x - 4}$.

6. 


   a) As best you can, copy this graph of a function $f$ defined only on the interval $[-1, 1]$ onto your paper (it is the lower half of the unit circle) and directly below it make a rough graph of the derivative function $y = f'(x)$, giving some explanation of how you constructed this graph.
   b) What is the domain of $f'$? What is the range of $f'$? Explain.