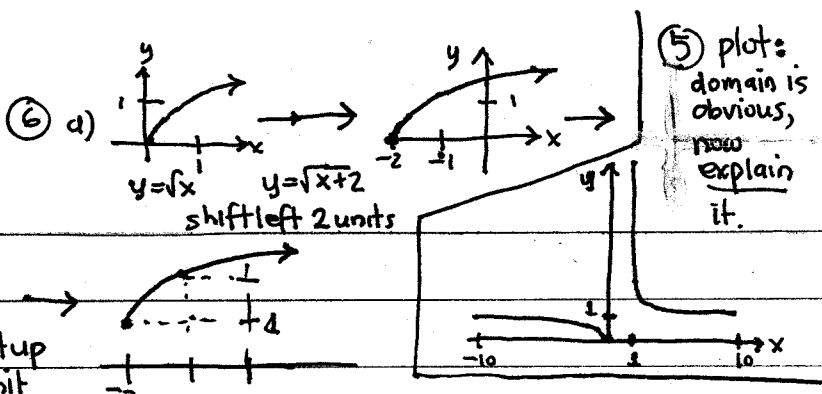
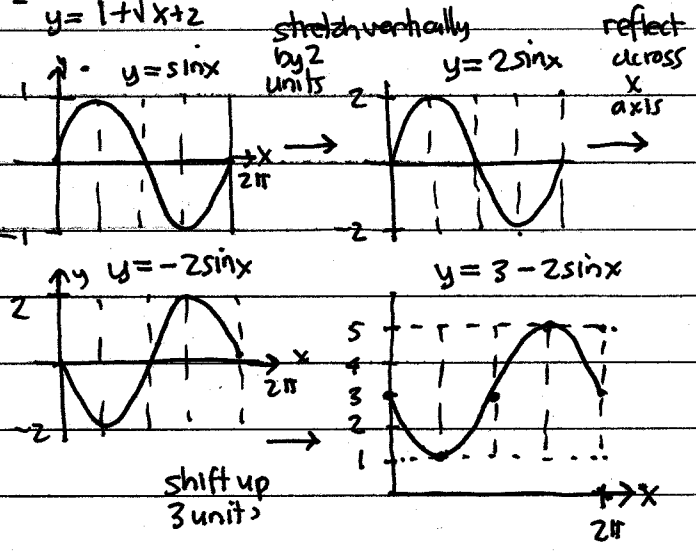


MAT1500-03/10 02F Test 1 Answers

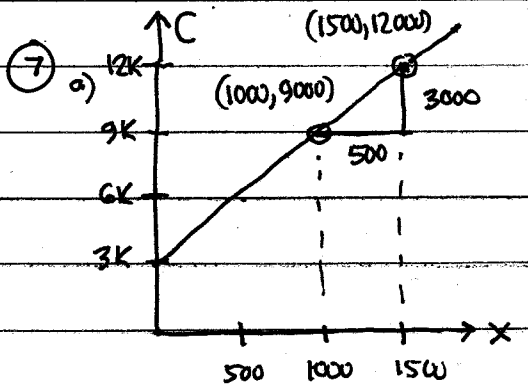
① $V = \frac{4\pi}{3} r^3$
 $S = 4\pi r^2 \rightarrow \frac{S}{4\pi} = r^2, \sqrt{\frac{S}{4\pi}} = r \geq 0$
 $V = \frac{4\pi}{3} \left(\frac{S}{4\pi}\right)^{3/2} = \frac{4\pi}{3} \left(\frac{S}{4\pi}\right)^{3/2}$
 $= \frac{4\pi}{3} \frac{S^{3/2}}{(4\pi)^{3/2}} = \frac{S^{3/2}}{6\pi^{1/2}}$
 so $V = \frac{S^{3/2}}{3(4\pi)^{1/2}} = \frac{S^{3/2}}{6\pi^{1/2}}$
 $p = 3/2$
 $C = \frac{1}{2} (C^{1/2})!$



② a) $y = 2 + 5(1 - e^{-x}) = f(x)$
 $y - 2 = 5(1 - e^{-x})$
 $\frac{y-2}{5} = 1 - e^{-x}$
 $\frac{y-2}{5} - 1 = -e^{-x} \rightarrow \frac{7-y}{5} = e^{-x}$
 $\ln\left(\frac{7-y}{5}\right) = \ln e^{-x} = -x$
 $x = -\ln\left(\frac{7-y}{5}\right) = \ln\left(\frac{7-y}{5}\right)^{-1} = \ln\frac{5}{7-y} = f^{-1}(y)$
 so $f^{-1}(x) = \ln\frac{5}{7-x}$



b) when $y = -3 = f(x)$, then $x = f^{-1}(-3)$
 $= \ln\frac{5}{7-(-3)} = \ln\frac{5}{10} = \ln\frac{1}{2} = \ln 2^{-1} = -\ln 2$
 (no need to repeat solving process)
 c) $f(\ln 2) = 2 + 5(1 - e^{-\ln 2}) = 2 + 5(1 - (e^{\ln 2})^{-1})$
 $= 2 + 5(1 - 2^{-1}) = 2 + 5(1 - \frac{1}{2}) = 2 + \frac{5}{2} = \frac{9}{2}$



③ $f(x) = 3^{\cos x^2} = g(h(j(x)))$
 a) $j(x) = x^2$
 $h(x) = \cos x$
 $g(x) = 3^x$
 b) $j(h(g(x))) = (\cos(3^x))^2$
 $= \cos^2 3^x$
 square

④ $\log_5 10 + \log_5 20 - 3\log_5 2 = \log_5 \left(\frac{200}{8}\right) = \log_5 25$
 $\log_5 \frac{10 \cdot 20}{200} - \log_5 \frac{2^3}{8} = \log_5 5^2 = 2\log_5 5 = 2$

$C = 9000 + 6x - 6000 = 3000 + 6x$

⑤ $f(x) = \sqrt{\frac{x}{x-1}}$ a) $\frac{x}{x-1}$ must be nonnegative and $x-1 \neq 0$
 so x and $x-1$ must have the same sign or $x=0$:
 $x > 0$ and $x-1 > 0$ or $x < 0$ and $x-1 < 0$
 $x > 1$ or $x < 0$
 $x > 1$ or $x < 0$ or $x = 0$

b) slope: 6 \$/toaster oven/week
 If costs 6 bucks to produce 1 TO, each week once production is started, ie for each additional TO.
 c) When $x=0$, $C=3000$
 If costs \$3000 a week before producing any toaster ovens at all ("Fixed costs")

so dom(f): $x \leq 0$ or $x > 1$
 or: $(-\infty, 0] \cup (1, \infty)$
 (or justify by checking signs on three intervals) $\text{sgn}\left(\frac{x}{x-1}\right) = \frac{+}{-} = -$ or $\frac{-}{-} = +$ (easiest)