Show all work, including mental steps, in a clearly organized way that speaks for itself. User proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. BOX final short answers. [See long instructions on reverse].

① The well known formulas $V = \frac{4}{3}\pi r^3$ and $S = 4\pi r^2$ for the volume and surface area of a sphere are power function relationships (i.e., of the form $y = cx^p$, where c and p are constants). Express the volume as a function of the surface area, giving your final result in this same power function form.

② a) If $y = 2 + 5(1 - e^{-x})$ defines a function $f(x)$, solve this relationship to find an expression for the inverse function $f^{-1}(x)$, and then simplify your result using algebra and rules of logarithms until it takes the simple form

$f^{-1}(x) = \ln\left(\frac{a}{b+cx}\right)$, where a, b, c are constants.

b) For what value of x does $y = -3$ in the above equation? (i.e. in part a))

c) Simplify $f(\ln 2)$ exactly using algebra to show that its value is $9/2$.

③ $f(x) = 3^{\cos(x^2)}$

a) Express f as the composition of three functions : $f(x) = g(h(j(x)))$ by stating formulas for $g(x)$, $h(x)$, and $j(x)$.

b) Evaluate $j(h(g(x)))$.

④ Show that the exact value of $\log_5 10 + \log_5 20 - 3\log_5 2$ is just 2 by combining these into a single logarithmic expression and simplifying it. Show each step in the process.

⑤ $f(x) = \sqrt{\frac{x}{x-1}}$ a) What is the domain of f ? Explain why. (Looking at a graph is insufficient.)

b) What is the smallest number in the range of f ? Explain.

⑥ State the series of geometric operations (translate up/down/left/right by __ units, horizontal/vertical stretch/compress by factor of __, reflection across x/y axis) that take you from the graph

a) $y = \sqrt{x}$ to $y = 1 + \sqrt{x+2}$

b) $y = \sin x$ to $y = 3 - 2\sin x$.

Make a rough sketch illustrating the start/stop/intermediate graphs.

⑦ A small appliance manufacturer finds that it costs $9000 to produce 1000 toaster ovens a week and $12,000 to produce 1500 toaster ovens per week.

a) Express the cost as a function of the number of toaster ovens produced weekly, assuming it is linear. First, draw a sketch locating the two data points given and the line they determine.

b) What is the slope of the graph and what does it represent ?

c) What is the "y intercept" of the graph and what does it represent ?