MAT1505-05/08 01S Quiz 8 Print Name (Last, First) ________________ /10

Show all work on this sheet, including indications of mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation/syntax. Label parts, $\boxed{\text{box}}$ final short answers.

Test each infinite series $\sum_{n=1}^{\infty} a_n$ for convergence or divergence, supporting your claim. Be convincing. Don't waste time.

(13) $\sum_{n=2}^{\infty} \frac{2}{n(\ln n)^3}$ [Note: $\int \frac{2}{n(\ln n)^3}\,dn = -\frac{1}{(\ln n)^2} + C$]

(23) $\sum_{n=1}^{\infty} (-1)^n 2^{\frac{1}{n}}$ (15) $\sum_{n=1}^{\infty} \frac{3^n n^2}{n!}$ (33) $\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{n\sqrt{n}}$

(13) $\int_2^{\infty} \frac{2}{x(\ln x)^3}\,dx = \lim_{a\to\infty} -\frac{1}{(\ln x)^2}\Big|_2^a = \lim_{a\to\infty}\left(-\frac{1}{(\ln a)^2} + \frac{1}{(\ln 2)^2}\right) = \frac{1}{(\ln 2)^2}$ converges

so by the integral test the original series $\boxed{\text{converges}}$

(23) $|a_n| = 2^{\frac{1}{n}}$ $\lim_{n\to\infty} |a_n| = \lim_{n\to\infty} 2^{\frac{1}{n}} = 2^0 = 1 \neq 0$, nth term does not go to zero so $\boxed{\text{diverges}}$ (divergence test)

(15) $a_n = \frac{3^n n^2}{n!}$

$a_{n+1} = \frac{3^{n+1}(n+1)^2}{(n+1)!}$

$$\frac{a_{n+1}}{a_n} = \frac{3^{n+1}(n+1)^2}{(n+1)!}\,\frac{n!}{3^n n^2} = \underbrace{\frac{3^{n+1}}{3^n}}_{3}\,\frac{(n+1)^2}{n^2}\,\frac{n!}{(n+1)!} \leftarrow (n+1)n!$$

$$= \frac{3}{n+1}\left(\frac{n+1}{n}\right)^2 = \frac{3}{n+1}\left(1+\frac{1}{n}\right)^2 \xrightarrow{n\to\infty} 0 < 1$$

effective ratio as $n\to\infty$ is zero so $\boxed{\text{converges}}$ (ratio test)

(33) $a_n = \frac{\tan^{-1} n}{n\sqrt{n}} \xrightarrow{n \text{ large}} \frac{\pi/2}{n^{3/2}}$ since $\lim_{n\to\infty} \tan^{-1} n = \frac{\pi}{2}$

this is a constant times a p-series with $p = 3/2 > 1$ so $\boxed{\text{converges.}}$ (limit comparison test)