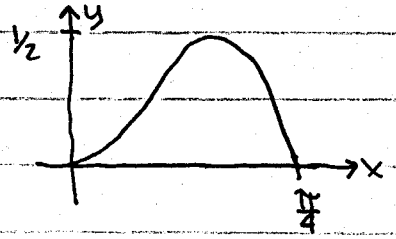
MAT1505-05/08 015 QUIZ 4 Print Name (Last, First) _____ /10

Show all work on this sheet, including indications of mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation/syntax. Label parts, box final short answers.



① a) Evaluate $\int x \sin 4x\,dx$

b) Use part a) to evaluate $\int_0^{\pi/4} x \sin 4x\,dx$.

c) Let $f(x) = x \sin 4x$. Write down an expression S_4 in terms of f for the Simpson rule $n=4$ approximation to this definite integral. [If you forgot the coefficients, give the $n=4$ midpoint approximation instead.]

d) Evaluate your expression from part c) using either MAPLE or your graphing calculator. Does your numerical result agree with part b)?

① a) $\int \underbrace{x}_{u} \underbrace{\sin 4x\,dx}_{dv} = \underbrace{x}_{u}\underbrace{\left(-\tfrac{1}{4}\cos 4x\right)}_{v} - \int \underbrace{\left(-\tfrac{1}{4}\cos 4x\right)}_{v}\underbrace{dx}_{du}$

$u = x$ $\quad dv = \sin 4x\,dx$

$\frac{du}{dx} = 1$ $\quad v = \int \sin 4x\,dx$ $\left(\to \int \sin w \frac{dw}{4}\right.$

$du = dx$ $\quad = -\frac{1}{4}\cos 4x$ $\left(\leftarrow = -\frac{1}{4}\cos w\right)$

$+ \frac{1}{4}\int \cos 4x\,dx$ $\left(\to \int \cos w \frac{dw}{4}\right.$

$= \frac{1}{16}\sin 4x$ $\left(\leftarrow = \frac{1}{4}\sin w\right)$

$= \boxed{-\frac{1}{4}x\cos 4x + \frac{1}{16}\sin 4x + C}$

b) $\int_0^{\pi/4} x\sin 4x\,dx = \left(-\frac{1}{4}x\cos 4x + \frac{1}{16}\sin 4x\right)\Big|_0^{\pi/4}$

$= -\frac{1}{4}\left(\frac{\pi}{4}\right)\underbrace{\cos\pi}_{-1} + \frac{1}{16}\underbrace{\sin\pi}_{0} - \left(-\frac{1}{4}\cdot 0\underbrace{\cos 0}_{0} + \frac{1}{16}\underbrace{\sin 0}_{0}\right) = \boxed{\frac{\pi}{16}}$

coefficients: 1 4 1 / 1 4 2 4 1 — all endpoints contribute

c) $\pi/32$ $3\pi/32$ $5\pi/32$ $7\pi/32$

$\Delta x = \frac{\pi/4}{4} = \frac{\pi}{16} = h$

$x_0 = 0$ $\frac{\pi}{16}$ $\frac{\pi}{8}$ $\frac{3\pi}{16}$ $\frac{\pi}{4} = x_4$

$$\boxed{S_4 = \frac{1}{3}\left(\frac{\pi}{16}\right)\left(f(0) + 4f\left(\tfrac{\pi}{16}\right) + 2f\left(\tfrac{\pi}{8}\right) + 4f\left(\tfrac{3\pi}{16}\right) + f\left(\tfrac{\pi}{4}\right)\right)}$$

```
> f:=x->x*sin(4*x);
d) > evalf(1/3*Pi/16*(f(0)+4*f(Pi/16)+2*f(Pi/8)+4*f(3*Pi/16)+f(Pi/4)));
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.1967971937

$$\left[M_4 = \frac{\pi}{16}\left(f\left(\tfrac{\pi}{32}\right) + f\left(\tfrac{3\pi}{32}\right) + f\left(\tfrac{5\pi}{32}\right) + f\left(\tfrac{7\pi}{32}\right)\right) \approx .2015\right]$$

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> evalf(Pi/16);
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.1963495409

error in 4th decimal place. pretty good.

Yes, results agree.

NOTE:
$\int \cos ax\,dx = \frac{1}{a}\sin ax + C$
$\int e^{ax}dx = \frac{1}{a}e^{ax} + C$
coefficient of function input variable multiplies derivative (chain rule)
divides antiderivative (variable substitution)