MAT1505-05/08 01S Final Exam Print Name (Last, First) ______ /100

See reverse side for instructions.

① Find the area of the region bounded by the parabolas $y=2x-x^2$ and $y=x^2$. First make a rough sketch of the situation, labeling the axes, appropriate tickmarks, and the curves by their equations, shading in the region whose area you are evaluating.

② Find the average value of $f(x)=x\cos 2x$ on the interval $[0,\frac{\pi}{2}]$.

③ Evaluate $\int_{-1}^{0}\frac{x}{x-1}\,dx$.

④ Re-express the integral $\int_{1}^{2}\frac{\sqrt{1+x^2}}{x}\,dx \approx 1.2220$ in terms of the new variable $z=1+x^2$, but do not evaluate it. [You can check your result in MAPLE against the given numerical value.]

⑤ Given that $\int e^{-x}\cos x\,dx = \frac{1}{2}e^{-x}(\cos x+\sin x)+C$;

a) Check that this is correct by differentiation.

b) Use it to evaluate $\int_{0}^{\infty} e^{-x}\cos x\,dx$.

⑥ a) If $v(t)=t-\sin t=\frac{ds(t)}{dt}$ is a velocity function, find the general displacement function $s(t)$.

b) Find the particular displacement function $s(t)$ which satisfies $s(0)=0$.

⑦ a) Evaluate $T_4(x)$, the Taylor series up to and including the fourth power term for $f(x)=\sqrt{1+x}$ centered about $x=3$.

b) Give the numerical values of each of these terms for $T_4(3.1)\approx\sqrt{4.1}$.

c) How many of these terms are required to achieve 5 decimal place accuracy? Explain in words.

d) Using parts b) and c), evaluate $\sqrt{4.1}$ to 5 decimal place accuracy.

⑧ Suppose $S(x)=\sum_{n=1}^{\infty}(-1)^n\frac{x^n}{n(2n+1)4^{n+1}}+C$. a) What value must C have for $S(0)=0$ to be satisfied?

b) Find the complete interval of convergence. Be sure to discuss convergence at its endpoints, justifying your claims.

⑨ $\frac{dy}{dt}=te^y$, $y(-1)=0$. a) Find the solution (in the form $y=y(x)$) of the differential equation that satisfies the initial condition.

b) Check that your result is valid by backsubstituting it into the differential equation and simplifying. Then check that your initial condition is satisfied by evaluating $y(-1)$.

c) At what values of t does your solution have vertical asymptotes?

d) Draw a rough directionfield for the 5x3 integer grid points $-2\le t\le 2$, $-1\le y\le 1$. Identify the initial condition point by "⊙" and draw in the solution curve through it.

e) Evaluate $y(0)$ numerically for your exact solution.

f) Use Euler's method with stepsize $h=1$ to approximate $y(0)$ starting with the initial condition $y(-1)=0$.

Math Exam Rules

READ THESE INSTRUCTIONS CAREFULLY

This test is not about just getting "the right answer", but also presenting and communicating well the process which leads to the results requested in each part of every problem, as well as your understanding of the course content and its vocabulary. [This is good practice for learning how to communicate technical results to other people in a workplace environment.] No results here may be justified using technology -- a reasoned explanation supported by mathematical facts is always required and cannot be substituted by a technology result. However, you are encouraged to use MAPLE to check every result you derive by hand. [For a takehome exam, no collaboration is allowed but you may consult your textbook, your notes and my handouts.] Come talk to me if you get stuck on any problem.

Show **all** work and answers, including indications of mental steps, on the lined paper provided. Put your name on each sheet and clearly label continuations of problems from one sheet to another. Label and SEPARATE clearly each part of each problem and BOX each short final response requested (and nothing else). Cross out abandoned work not to be considered.

Use proper mathematical notation: "symbol" = "expression representing symbol" = ... Don't misuse equal signs, and don't write down unidentified expressions, but do link expressions which are equal with equal signs. Give EXACT ANSWERS, not decimal approximations, unless the context warrants it, but first give the exact result in any case. Always simplify results.

When you have completed the exam, please read and sign the dr bob integrity pledge:

"During this examination, all work has been my own. I give my word as a decent human being that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated." (I have not opened any software other than MAPLE.)

Signature:

Date: