MATH500-05/08 01S TEST 3 Takehome Print Name (Last, First) _____ /100
see instructions at end.

① a) Obtain the complete Taylor series for $\ln x$ centered at $x=1$ directly from the Taylor series formula.
b) Write out the first 5 nonzero terms.
c) Use your result to evaluate $\ln 1.1$ to 4-decimal place accuracy, based on the alternating series estimate for the error.

② a) Evaluate $\int \frac{1}{x \ln x}\,dx$ and $\int \frac{1}{x(\ln x)^p}\,dx$ where $p>1$, using variable substitution.
b) For what values of $p \geq 1$ does the series $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$ converge? Explain.

③ Evaluate: a) $\int_0^a x e^{-2x^2}\,dx$ and $\int_0^\infty x e^{-2x^2}\,dx$. Use careful limit notation.
b) $\int_0^a x e^{-2x}\,dx$ and $\int_0^\infty x e^{-2x}\,dx$.

④ a) Given $f(x) = \frac{1}{1-\frac{x}{2}}$, evaluate and simplify $g(x) = x f'(x)$.
b) Obtain the power series for $f(x)$ from the power series $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$, $|x|<1$. What is the interval of convergence for the new series? Justify your answer. Does 1 belong to this interval?
c) Now obtain the power series for $g(x)$ by term by term ~~multiplication~~ differentiation and then multiplication by x. What is its interval of convergence?
d) Now evaluate $g(1)$ from your formula of part a) and evaluate the power series for $g(x)$ at $x=1$ whose sum is just $g(1)$, leading to an explicit value for the resulting infinite series. State the infinite series and its sum.
[Repeating this procedure two more times leads to the exact value $\sum_{n=1}^{\infty} \frac{n^3}{2^n} = 26$ we found doing Calclab problem 8.7.9, which inspired me to understand why.]

⑤ Stewart 8.1.35. A hawk flying at 15 m/s at an altitude of 180 m accidentally drops its prey. The parabolic trajectory of the falling prey is described by the equation $y = 180 - \frac{x^2}{45}$, until it hits the ground, where y is its height above the ground and x is the horizontal distance traveled by the prey from the time it is dropped until the time it hits the ground. Find actual distance traveled by the prey during this fall.
a) Give the exact integral formula.
b) Use Simpson's rule with $n=4$ to approximate it, showing carefully all of your steps.

⑥ Stewart 8.R.18.

t	0	2	4	6	8	10	12	14	16	18	20	22	24
c(t)	0	1.9	3.3	5.1	7.6	7.1	5.8	4.7	3.3	2.1	1.1	0.5	0

After a 6-mg injection of dye into the heart, the readings of dye concentration at two second intervals are given by this table. Use Simpson's rule to approximate $\int_0^{24} C(t)\,dt$.

⑦ Find the average value of the function $f(x) = x^2\sqrt{1+x^3}$ on the interval $[0,2]$. (6.R.50)

⑧ Stewart 5.R.58. A particle moves along a line with velocity function $v(t) = t^2 - t$. Find a) the displacement and b) the distance traveled by the particle during the time interval $[0,5]$.