

Show all work, including indications of mental steps, in a clearly organized presentation labeled by problem number and part and box short final answers requested in each part. Use proper mathematical notation/syntax: use symbols to identify all expressions and use equal signs whenever appropriate, or arrows indicating a relation which is not an equality.

- ① a) Derive the complete Taylor series (at $x=0$ center) for $f(x) = e^{-x}$ directly from the definition.
 b) Write out the first 5 terms explicitly, and indicate their individual numerical values for $f(0.1) = e^{-0.1}$
 c) How many terms are needed for 4 decimal place accuracy?
 d) Evaluate $e^{-0.1}$ to 4 decimal place accuracy, showing your calculations.
 e) What is the Taylor series (centered at $x=0$) for $g(x) = xe^{-x}$?

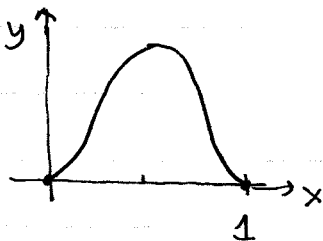
- ② $\sum_{n=0}^{\infty} \frac{2^n (x-3)^n}{\sqrt{n+3}}$ a) Find the radius of convergence R and
 b) the interval of convergence (i.e. $1 \leq x < 2$) checking the endpoint values for convergence to be sure of the complete interval of convergence.

- ③ $\sum_{n=1}^{\infty} n e^{-n^2}$ Test this series for convergence using the integral test. (Carefully state limits in evaluating the appropriate improper integral.)

- ④ Explain briefly why each of the following series converges or diverges:

a) $\sum_{n=1}^{\infty} \frac{1}{n^4 + n}$ b) $\sum_{n=1}^{\infty} \frac{n^2}{1+2n^2}$ c) $\sum_{n=1}^{\infty} (\sqrt{2})^n$

- ⑤ $f(x) = 30x^2(1-x)^2$ is a probability density function for the interval $0 \leq x \leq 1$ so its integral over that interval is 1. If you evaluate this integral with Simpson's rule using $n=4$ divisions, what is the percentage error in your approximation? Show how you get all the terms you add together to get this numerical approximation S_4 ; don't just state the result.



Recall: $\int_a^b f(x) dx = \frac{\Delta x}{3} [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \dots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$

After completing this exam date and sign this pledge:

I give my word that I have opened no software other than MAPLE on my computer and have engaged in no behavior that could even be suggestive of unethical behavior in regard to this exam, nor will I afterward with other students who have yet to take it.

signature: _____

date: _____