# MAT1505-05/08 OIS TEST 1 Answers

① $\displaystyle\int_1^9 \frac{3x}{\sqrt{10-x}}\,dx = \int_{x=1 \to u=9}^{x=9\to u=1} \frac{3x}{\sqrt{u}}(-du) = \int_9^1 \frac{3(10-u)(-du)}{u^{1/2}}$

$$\left(\begin{array}{ll} u=10-x & x=10-u \\ \frac{du}{dx}=-1 & \\ du=-dx & \\ -du=dx & \end{array}\right)$$

$= -3\int_9^1 10u^{-1/2} - u^{1/2}\,du$

$= -3\left(10\frac{u^{1/2}}{1/2} - \frac{u^{3/2}}{3/2}\right)\Big|_9^1$

$= -3\left(20u^{1/2} - \frac{2}{3}u^{3/2}\right)\Big|_9^1 = -3\left(\left(20-\frac{2}{3}\right) - \left(20\, 9^{1/2} - \frac{2}{3}9^{3/2}\right)\right)$

$= -3\left(20 - \frac{2}{3} - \left(20\cdot 3 - \frac{2}{3}\cdot 3^3\right)\right) = -60 + 2 + 3(\underbrace{60-18}_{42})$

$= 128 - 60 = \boxed{68}$

④ b) $V_{avg}(\text{halfcycle}) = \dfrac{\displaystyle\int_{-\frac{1}{240}}^{\frac{1}{240}} V_0\cos(120\pi t)\,dt}{\frac{1}{240} - \left(-\frac{1}{240}\right)}$

$= 120\displaystyle\int_{-\frac{1}{240}}^{\frac{1}{240}} V_0 \cos 120\pi t\,dt$

$= 120\,\dfrac{V_0}{120\pi}\sin(120\pi t)\Big|_{-1/240}^{1/240}$

$= \dfrac{V_0}{\pi}\left(\underbrace{\sin\frac{\pi}{2}}_{1} - \underbrace{\sin\left(-\frac{\pi}{2}\right)}_{-1}\right) = \boxed{\dfrac{2V_0}{\pi}}$

② a) $\displaystyle\int \underbrace{x}_{u}\underbrace{e^{-\frac{x}{2}}dx}_{dv} = \underbrace{x}_{u}\underbrace{(-2e^{-\frac{x}{2}})}_{v} - \int \underbrace{(-2e^{-\frac{x}{2}})}_{v}\underbrace{dx}_{du}$

$$\left(\begin{array}{ll} u=x & dv = e^{-\frac{x}{2}}dx \\ \frac{du}{dx}=1 & v=\int e^{-\frac{x}{2}}dx \\ du=dx & \quad = \frac{e^{-\frac{x}{2}}}{-1/2} = -2e^{-\frac{x}{2}} \end{array}\right)$$

$= -2xe^{-\frac{x}{2}} + 2\underbrace{\int e^{-\frac{x}{2}}dx}_{\frac{e^{-x/2}}{-1/2}}$

$= \boxed{-2xe^{-x/2} - 4e^{-x/2} + C = -2e^{-x/2}(x+2) + C}$

b) $\displaystyle\int_0^4 xe^{-x/2}\,dx = -2xe^{-x/2} - 4e^{-x/2}\Big|_0^4$

$= -8e^{-2} - 4e^{-2} - (0 - 4\cdot 1) = \boxed{4 - 12e^{-2}}$

⑤ $\displaystyle\int_0^1 \underbrace{x}_{f(x)}\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} \underbrace{f(x_i)}_{x_i}\Delta x$

$$\left(\begin{array}{l} \Delta x = \frac{1-0}{n} = \frac{1}{n} \\ x_i = 0 + i\Delta x = \frac{i}{n} \end{array}\right)$$

$= \lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{i}{n}\right)\left(\frac{1}{n}\right)$

$= \lim_{n\to\infty}\sum_{i=1}^{n}\frac{i}{n^2} = \lim_{n\to\infty}\frac{1}{n^2}\sum_{i=1}^{n} i$

$= \lim_{n\to\infty}\frac{1}{n^2}\frac{n(n+1)}{2} = \lim_{n\to\infty}\frac{1}{2}\left(\frac{n+1}{n}\right)$

$= \lim_{n\to\infty}\frac{1}{2}\left(1+\underbrace{\frac{1}{n}}_{\to 0}\right) = \boxed{\frac{1}{2}}$

③ $\displaystyle\int_1^3 \frac{2\,dx}{2x-3}$ where zero? at $x = 3/2 \in [1,3]$ vertical asymptote — cannot use antiderivative evaluation

$\displaystyle\int \frac{2\,dx}{2x-3} = \int\frac{du}{u} = \ln|u| + C = \ln|2x-3| + C$

$u = 2x-3$

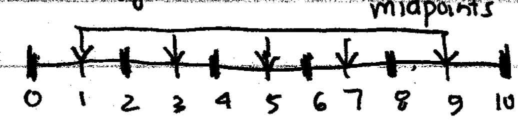
$\frac{du}{dx} = 2$

$du = 2dx$

BUT $\displaystyle\int_1^3 \frac{2\,dx}{2x-3} \neq \ln|2x-3|\Big|_1^3 = \ln 3 - \ln 1 = \ln 3$

⑥ $s = \displaystyle\int_0^{10} v(t)\,dt$

midpoints



$\Delta t = 2$ min (5 divisions to have midpoint data)

$s \approx 2(v(1)+v(3)+v(5)+v(7)+v(9))$

$= 2(42+49+54+57+55)\frac{mi}{hr}$

$= 2(257)\frac{mi}{hr} = \frac{2(257)}{60}$ mi

last minute reread of problem: s → min

$\frac{mi}{60 \text{ min}}$ $\approx \boxed{8.57 \text{ mi}}$

when units are important, just keep them in the calculation

④ a) $V_{avg}(1\text{sec}) = \frac{1}{1}\displaystyle\int_0^1 V_0\cos(120\pi t)\,dt = \frac{V_0}{120\pi}\sin(120\pi t)\Big|_0^1$

$\displaystyle\int V_0\cos(\underbrace{120\pi t}_{u})\,dt = \int V_0\cos u\,\frac{du}{120\pi} = \frac{V_0}{120\pi}\int\cos u\,du$

$u = 120\pi t$

$\frac{du}{dt} = 120\pi$

$du = 120\pi\,dt$

$\frac{du}{120\pi} = dt$

$= \frac{V_0}{120\pi}\sin u + C = \frac{V_0}{120\pi}\sin(120\pi t) + C$

$= \frac{V_0}{120\pi}\left(\underbrace{\sin(120\pi)}_{\sin(60(2\pi))=0} - \underbrace{\sin 0}_{0}\right) = \boxed{0}$