MAT 2705 005 Quiz 6 Print Name (Last, First) ________________ /10

Show absolutely all work (no scratch paper calculations omitted or mental calculations unreported) on this sheet in a clearly organized way, labeling problems, parts, and expressions (by their proper symbols). $\boxed{\text{Box}}$ short final answers.

$\vec{b}_1 = (1,-2)$, $\vec{b}_2 = (1,1)$, $\vec{v} = (1,2)$, $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \dfrac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

a) Make a diagram (label axes) of the vectors $\vec{b}_1, \vec{b}_2, \vec{v}$ and including the parallelogram representing the projections along the axes of $\vec{b}_1$ and $\vec{b}_2$ of the vector $\vec{v}$.

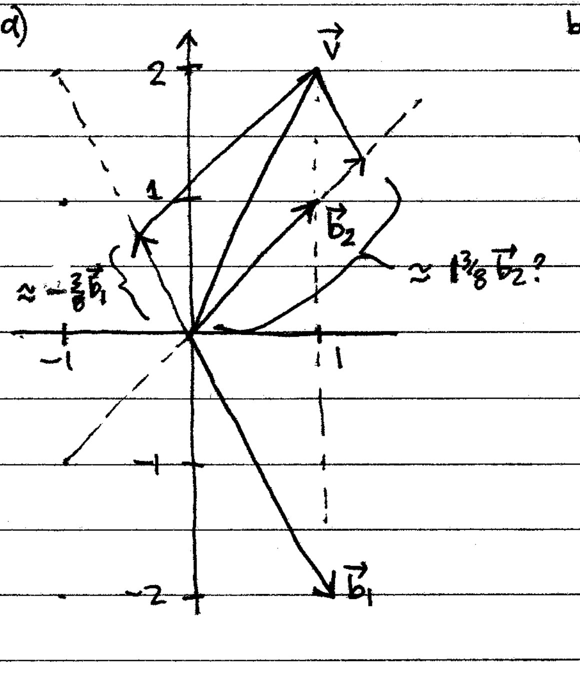
b) From your diagram, guess approximate (rough) values of the coordinate values y_1 and y_2 for $\vec{v}$ expressed in the basis $\{\vec{b}_1, \vec{b}_2\}$.

c) Now find these coordinates using either an inverse matrix or row reduction
[final answer: $y_1 = \dots$, $y_2 = \dots$]

d) How close was your guess? (Dont change your guess now!).

e) Now find the coordinates (y_1, y_2) of a general vector $\vec{x} = (x_1, x_2)$ with respect to this basis. [final answer: $y_1 = \dots$, $y_2 = \dots$]

a)



b) Looks roughly like $\boxed{y_1 \approx -3/8,\ y_2 \approx 1\,3/8}$

c) $\vec{v} = y_1\vec{b}_1 + y_2\vec{b}_2$

$$\begin{bmatrix}1\\2\end{bmatrix} = y_1\begin{bmatrix}1\\-2\end{bmatrix} + y_2\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}1 & 1\\-2 & 1\end{bmatrix}\begin{bmatrix}y_1\\y_2\end{bmatrix}$$

solve:

$$\begin{bmatrix}y_1\\y_2\end{bmatrix} = \begin{bmatrix}1 & 1\\-2 & 1\end{bmatrix}^{-1}\begin{bmatrix}1\\2\end{bmatrix} = \frac{1}{3}\begin{bmatrix}1 & -1\\2 & 1\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix} = \frac{1}{3}\begin{bmatrix}-1\\4\end{bmatrix} = \begin{bmatrix}-1/3\\4/3\end{bmatrix}$$

$$\frac{1}{3}\begin{bmatrix}1 & -1\\2 & 1\end{bmatrix}$$

$\boxed{y_1 = -1/3,\ y_2 = 4/3}$

d) not bad for a rough diagram.

e) $$\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}1 & 1\\-2 & 1\end{bmatrix}\begin{bmatrix}y_1\\y_2\end{bmatrix} \rightarrow \begin{bmatrix}y_1\\y_2\end{bmatrix} = \frac{1}{3}\begin{bmatrix}1 & -1\\2 & 1\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}(x_1-x_2)/3\\(2x_1+x_2)/3\end{bmatrix}$$

$$\boxed{y_1 = \frac{x_1-x_2}{3},\quad y_2 = \frac{2x_1+x_2}{3}}$$

General approach: extend $\vec{b}_1$ and $\vec{b}_2$ to their "axes", then draw in lines parallel to $\vec{b}_1$ and $\vec{b}_2$ through the tip of $\vec{v}$ until they cross the "axes".