MAT 2705 005 Quiz 4 Print Name (Last, First) jantzen, bob /10

Show absolutely all work (no scatch paper calculations or mental calculations unreported) on this sheet in a clearly organized way, labeling problems, parts and expressions written down.

① $3x_1+5x_2+x_3=3$
$x_1+2x_2+x_3=1$

a) Write this system in the vector (matrix) form $A\vec{x}=\vec{b}$.

b) Let $B=[A,\vec{b}]$ be the augmented matrix. Show each step in the reduction of B to its "rref form" rref(B).

c) Write out the corresponding scalar equations, identifying the bound and free variables.

d) Find the solution of the system.

② $B=\begin{bmatrix}0&1&1&0&1&0&1\\0&0&0&1&1&0&1\\0&0&0&0&0&1&1\end{bmatrix}$ a) If this is the rref form of the augmented matrix of a linear system, write out the corresponding equivalent scalar equations and identify bound and free variables.

b) Find the solution of the system.

① a) $\underbrace{\begin{bmatrix}3&5&1\\1&2&1\end{bmatrix}}_{A}\underbrace{\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}}_{\vec{x}}=\underbrace{\begin{bmatrix}3\\1\end{bmatrix}}_{\vec{b}}$

b) $B=\begin{bmatrix}3&5&1&3\\1&2&1&1\end{bmatrix}\xrightarrow{\text{swaprow}(B,1,2)}\begin{bmatrix}1&2&1&1\\3&5&1&3\\ {\scriptstyle -3}&{\scriptstyle -6}&{\scriptstyle -3}&{\scriptstyle -3}\end{bmatrix}\xrightarrow{\text{addrow}(\%,1,2,-3)}\begin{bmatrix}1&2&1&1\\0&-1&-2&0\end{bmatrix}\xrightarrow{\text{mulrow}(\%,2,-1)}\begin{bmatrix}1&2&1&1\\ {\scriptstyle 0}&{\scriptstyle -2}&{\scriptstyle -4}&{\scriptstyle 0}\\0&1&2&0\end{bmatrix}$

$\xrightarrow{\text{addrow}(\%,1,2,-2)}\begin{bmatrix}1&0&-3&1\\0&1&2&0\end{bmatrix}=\text{rref}(B)$

c) $\begin{array}{cccc}B&B&F&\\x_1&x_2&x_3=t_1&\end{array}$

$\begin{bmatrix}1&0&-3&1\\0&1&2&0\end{bmatrix}$

$x_1-x_3=1$ $\quad x_1=1+3x_3=1+3t_1$

$x_2+2x_3=0$ $\quad x_2=-2x_3=-2t_1$

$\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}1+3t_1\\-2t_1\\t_1\end{bmatrix}$ solution

or $\vec{x}=(1+3t_1,-2t_1,t_1)$

② $\begin{array}{ccccccc}F&B&F&B&F&B&\\x_1&x_2&x_3&x_4&x_5&x_6&\end{array}$

$\begin{bmatrix}0&1&1&0&1&0&1\\0&0&0&1&1&0&1\\0&0&0&0&0&1&1\end{bmatrix}$

$x_2+x_3+x_5=1 \rightarrow x_2=1-x_3-x_5=1-t_2-t_3$

$x_4+x_5=1 \qquad x_4=1-x_5=1-t_3$

$x_6=1 \qquad x_6=1 \qquad =1$

bound

free: x_1, x_3, x_5
$\quad\;\; = t_1,\ t_2,\ t_3$

$\begin{bmatrix}x_1\\x_2\\x_3\\x_4\\x_5\\x_6\end{bmatrix}=\begin{bmatrix}t_1\\1-t_2-t_3\\t_2\\1-t_3\\t_3\\1\end{bmatrix}$ solution

or $\vec{x}=(t_1,1-t_2-t_3,t_2,1-t_3,t_3,1)$

→ zero column means variable does not appear in system so it can take any value (it is a free variable)
[a zero column does not mean the corresponding variable is zero]

→ without labeling columns by variables, writing B or F by a column does not tell me which variables are bound and free