

Show absolutely all work (no scratch paper calculations unreported) on this sheet in a clearly organized way, labeling problems, parts and expressions written down.

- ① Write down a typical example of
- a 3×3 upper triangular matrix
 - a diagonal 2×2 matrix.
- ② Write out explicitly the identity matrix I_3 and the zero matrix O_{31} .
- ③ Evaluate a) $2A - B$, b) AB for $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$.
- ④ $B = \begin{bmatrix} 2 & -3 & 4 & 13 \\ 1 & 1 & -1 & -2 \\ 5 & 4 & 1 & 3 \end{bmatrix}$ is the augmented matrix of a linear system of equations in the variables X_i .
- Write out the (scalar) equations explicitly.
 - Show that the triple $(1, -1, 2)$ is a solution of this system.

① a) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$ (zero entries below main diagonal) b) $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ (zeros off main diagonal)

② $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (1s on main diagonal) $O_{31} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ (3 rows, 1 column)

③ a) $2A - B = 2 \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} - \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 2-4 & 4-3 \\ 6-2 & 8-1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 4 & 7 \end{bmatrix}$

b) $AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1(4)+2(2) & 1(3)+2(1) \\ 3(4)+4(2) & 3(3)+4(1) \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ 20 & 13 \end{bmatrix}$

④ a) $2X_1 - 3X_2 + 4X_3 = 13$
 $X_1 + X_2 - X_3 = -2$
 $5X_1 + 4X_2 + X_3 = 3$

b) $2(1) - 3(-1) + 4(2) = 2 + 3 + 8 = 13 \checkmark$
 $1(1) + (-1) - (2) = -2 \checkmark$
 $5(1) + 4(-1) + (2) = 5 - 4 + 2 = 3 \checkmark$

or $\begin{bmatrix} 2 & -3 & 4 \\ 1 & 1 & -1 \\ 5 & 4 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2+3+8 \\ 1-1-2 \\ 5-4+2 \end{bmatrix} = \begin{bmatrix} 13 \\ -2 \\ 3 \end{bmatrix} \checkmark$