MAT2705-005 Quiz2 Print Name (Last, First) jantzen, bob /10

Label and clearly separate each part of each problem, $\boxed{\text{boxing}}$ any final results requested if appropriate. Organize your presentation so that it speaks for itself.

$$y' + 2y = 5e^{-3t},\quad y(0) = 0$$

5 a) Find the general solution using the linear solution algorithm.

2 b) Find the particular solution which satisfies the initial condition.

1 c) Show that your particular solution satisfies the differential equation.

1 d) Find the value t_{max} at which y is a maximum.

1 e) Evaluate $y_{max} = y(t_{max})$ and simplify to a rational number or at least a decimal approximation.

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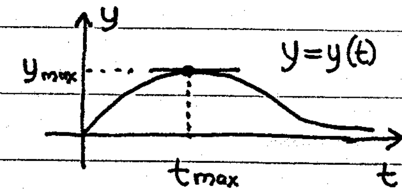

a) $e^{2t}[y' + 2y = 5e^{-3t}] \to e^{2t}(y' + 2y) = 5e^{2t}e^{-3t}$

$e^{\int 2\,dt = 2t} \quad e$

$$\frac{d}{dt}(ye^{2t}) = 5e^{-t}$$

$$ye^{2t} = \int 5e^{-t}\,dt = 5\left(\frac{e^{-t}}{-1}\right) + C_1 = -5e^{-t} + C_1$$

general soln: $\boxed{y = e^{-2t}(-5e^{-t} + C_1) = -5e^{-3t} + C_1 e^{-2t}}$

b) $0 = y(0) = -5\cdot 1 + C_1\cdot 1 \to C_1 = 5$, so $\boxed{y = 5e^{-2t} - 5e^{-3t} = 5(e^{-2t} - e^{-3t})}$

c) $y = 5(e^{-2t} - e^{-3t})$

$y' = 5(-2e^{-2t} + 3e^{-3t})$

$y' + 2y = 5(-2e^{-2t} + 3e^{-3t}) + 5(2e^{-2t} - 2e^{-3t}) = 5e^{-3t}$ ✓

$3-2=1$

d) Find value of t where slope zero:

$0 = y' = 5(-2e^{-2t} + 3e^{-3t}) \to -2e^{-2t} + 3e^{-3t} = 0 \to [2e^{-2t} = 3e^{-3t}]\,\frac{e^{3t}}{2}$

$\to e^{3t}e^{-2t} = \frac{3}{2} \to e^{t} = \frac{3}{2} \quad t = \ln e^{t} = \boxed{\ln\frac{3}{2} = t_{max}} \quad \approx 0.405$

e) $y_{max} = y(t_{max}) = y(\ln\frac{3}{2}) = 5(e^{-2\ln\frac{3}{2}} - e^{-3\ln\frac{3}{2}})$

$= 5((e^{\ln\frac{3}{2}})^{-2} - (e^{\ln\frac{3}{2}})^{-3})$

$= 5[(\frac{3}{2})^{-2} - (\frac{3}{2})^{-3}]$

$= 5[(\frac{2}{3})^{2} - (\frac{2}{3})^{3}] = 5(\frac{2}{3})^{2}[1 - \frac{2}{3}] = \boxed{\frac{20}{27}} \approx .74$