

You may check any of your work with MAPLE or a graphing calculator BUT your responses to every problem should be presented as though you had no access to technology. Document every step you take including those which may be obvious to you but which help another person follow your work without having to do mental calculations or guess what an expression represents (not identified by an appropriate symbol). Add explanatory comments if they help. Box final answers requested by each part (and nothing else). Clearly label each part.

① a) $3 \frac{dx}{dt} + 2x = e^{-t/2}$ (i) Find the general solution using the appropriate algorithm.
 $x(0) = 0$ (ii) Find the solution satisfying the initial condition.

b) bob, chuck, and dave are sitting around drinking Starbucks bottled Frappuccinos when one of their students asks them to come up with solutions to the DE

$\frac{dy}{dt} = \frac{y+1}{t+1}$. After much discussion, bob says $y(t) = t$, chuck says $y(t) = 2t+1$, and dave says $y(t) = t^2 - 2$. Who is right? Respond with the following steps:

(i) Find the general solution to this DE, using the separable technique.

(ii) Find the three solutions satisfying the respective initial conditions $y(0) = 0$, $y(0) = 1$, $y(0) = -2$ which correspond to their responses above.

(iii) Now answer the question, who is right?

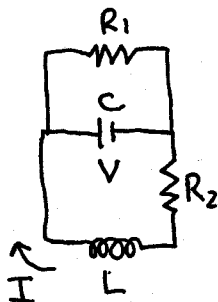
(iv) For one of the right answers, check that it is indeed a soln by backsubstituting into the DE and showing that it is satisfied.

② $y'' + 3y' + 2y = e^{-t/2}$ a) Find the general solution.
 $y'(0) = 0 = y(0)$ b) Find the solution satisfying the initial conditions.

③ $A = \begin{bmatrix} 3 & 0 & 1 & 0 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$ a) Using only matrix multiplication, check whether \vec{u} or \vec{v} is an eigenvector of A , and if so identify the corresponding eigenvalue.
 b) One of these 2 vectors does belong to an eigenspace of A , with dimension greater than 1. Knowing its eigenvalue from part a), find a basis for this eigenspace only. What is the value of the dimension of this particular eigenspace?
 $\vec{u} = (1, 2, 3, 0)$
 $\vec{v} = (3, 3, -3, -2)$

④ $A = \begin{bmatrix} -8 & -12 & -6 \\ 2 & 1 & 2 \\ 7 & 12 & 5 \end{bmatrix}$ $\text{eigenvecs}(A) = [-2, 1, \{[-1, 0, 1]\}], [-1, 1, \{[-6, 1, 5]\}], [1, 1, \{[-1, 1, 4]\}]$
 a) Use this information (no need to rederive it!) to find the general soln of the vector DE, after writing out the system of scalar DE's and initial conditions that it represents.
 b) Find the solution which satisfies the initial condition and give your final answer in scalar form.
 $\vec{x}' = A\vec{x}$
 $\vec{x}(0) = (1, 0, 0)$

⑤



I = current
 V = voltage drop
 across capacitor

$R_1 = 5 \text{ ohm}$
 $R_2 = .8 \text{ ohm}$
 $C = .1 \text{ farad}$
 $L = .4 \text{ henry}$

$$\begin{bmatrix} I' \\ V' \end{bmatrix} = \begin{bmatrix} -R_2/L & -1/L \\ 1/C & -1/(R_1 C) \end{bmatrix} \begin{bmatrix} I \\ V \end{bmatrix}$$

$$I(0) = 3 \text{ amp} , V(0) = 3 \text{ volts}$$

By introducing the vector $\vec{X} = \begin{bmatrix} I \\ V \end{bmatrix}$
 and evaluating the coefficients, this circuit

DE system reduces to the mathematical problem :

$$\vec{X}' = A\vec{X}, \vec{X}(0) = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, A = \begin{bmatrix} -2 & -5/2 \\ 10 & -2 \end{bmatrix}$$

- Find the eigenvalues and eigenvectors $\{\lambda_i\}$ and $\{\vec{b}_i\}$ of the coefficient matrix A .
- Find the real and imaginary parts of the complex eigenvector solutions $\vec{X}_1 + i\vec{X}_2 = e^{\lambda_1 t} \vec{b}_1$.
- Find the general solution of the DE system.
- Find the solution which satisfies the initial conditions and state your final answer in scalar form $x_1 = \dots, x_2 = \dots$

When you have completed the exam, please read and sign the dr bob integrity pledge if it applies:

"During this examination, all work has been my own and I have not opened any software other than MAPLE on my computer. I give my word as a decent human being that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination."

Signature :

Date :