Test 3

Mat 2705-02 00S Takehome Exam Rules PrintName (Last, First) ______________________

READ THESE INSTRUCTIONS CAREFULLY

This test is not about just getting "the right answer", but also presenting and communicating well the process which leads to the results requested in each part of every problem, as well as your understanding of the course content and its vocabulary. [This is good practice for learning how to communicate technical results to other people in a workplace environment.] No results here may be justified using technology -- a reasoned explanation supported by mathematical facts is always required and cannot be substituted by a technology result.* However, you are encouraged to use MAPLE to check every result you derive by hand. No collaboration is allowed but you may consult your textbook, your notes and my handouts. Come talk to me if you get stuck on any problem.

Show **all** work and answers, including indications of mental steps, on the lined paper provided. Put your name on each sheet and clearly label continuations of problems from one sheet to another. Label and SEPARATE clearly each part of each problem and BOX each short final response requested (and nothing else). Cross out abandoned work not to be considered.

Use proper mathematical notation: "symbol" = "expression representing symbol" = ... Don't misuse equal signs, and don't write down unidentified expressions, but do link expressions which are equal with equal signs. Give EXACT ANSWERS, not decimal approximations.

When you have completed the exam, please read and sign the dr bob integrity pledge:

"During this examination, all work has been my own. I give my word as a decent human being that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination."

Signature:

Date:

① $A = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \end{bmatrix}$

a) Find a basis $\{\vec{b}_1, \ldots\}$ for the nullspace of A.

b) What is the dimension of nullspace A? why?

c) Does $[4,-7,2,1]$ belong to this space? Explain why or why not. If it does, find its coordinates (components) with respect to your basis.

② $\vec{v}_1 = [1,1,-1,-1]$

$\vec{v}_2 = [2,-1,3,1]$

$\vec{v}_3 = [-1,2,-4,-2]$

$\vec{v}_4 = [1,1,2,1]$

$\vec{v} = [6,3,5,1]$

a) Express $\vec{v}$ as a linear combination of the vectors $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$ if possible.

[your final answer should be in the form $\vec{v} = (\ldots)\vec{v}_1 + (\ldots)\vec{v}_2 + \ldots$ if possible]

b) Is $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$ a basis of $\mathbb{R}^4$? Explain.

c) What is the dimension of span$\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$? Explain.

③ $A = \begin{bmatrix} 3 & -2 & 3 & -2 \\ -2 & 3 & -2 & 3 \\ 3 & -2 & 3 & -2 \\ -2 & 3 & -2 & 3 \end{bmatrix}$

a) If it is known that A is diagonalizable and has eigenvalues $\{0, 2, 10\}$, find a basis $\{\vec{b}_1, \vec{b}_2, \vec{b}_3, \vec{b}_4\}$ of $\mathbb{R}^4$ consisting of eigenvectors of A.

b) Based on your work for a), state the multiplicities of these eigenvalues and explain your conclusion.

c) Let $B = \text{augment}(\vec{b}_1, \vec{b}_2, \vec{b}_3, \vec{b}_4)$ be the basis changing matrix. Evaluate B^{-1} with a row reduction process, indicating each row operation used.

d) Evaluate $B^{-1}AB$ by explicit matrix multiplication to show that your basis really does diagonalize A.

e) Find the new coordinates (components) of $[2, 4, 6, 8]$ with respect to your basis.

f) Does the vector $[1, -1, -1, 1]$ belong to an eigenspace of the matrix A? Explain.

④ $A = \begin{bmatrix} -2 & 4 \\ -3 & -2 \end{bmatrix}$

a) Find the eigenvalues of A.

b) Find linearly independent eigenvectors for each eigenvalue.

⑤
$$\begin{aligned} x_1' &= x_1 - 2x_2 + x_3 & \quad x_1(0) &= 2 \\ x_2' &= -2x_1 + x_2 + x_3 & \quad x_2(0) &= 3 \\ x_3' &= 3x_3 & \quad x_3(0) &= 2 \end{aligned}$$

Find the general solution and IVP solution by following these steps:

a) Write this system in vector (matrix) form: $\vec{x}' = A\vec{x}$, $\vec{x}(0) = \ldots$

b) Find the eigenvalues and eigenvectors of A.

c) Introduce a linear change of variables from $\vec{x}$ to $\vec{y}$ which diagonalizes A and obtain the equivalent DE satisfied by $\vec{y}$.

d) Write out the scalar form of the DE satisfied by $\vec{y}$ and solve these.

e) Use your result to find the general solution for $\vec{x}$ expressed in vector form, so that each of its components are obvious: $\vec{x} = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$

f) Find the solution which satisfies the initial conditions.

* MAPLE exception. To save time you may use rref in MAPLE to row reduce any matrix as long as you footnote your result and append the input matrix and result printout for all such occurrences. Clearly label them.