Lunar Lander Problem E&P2, Example 1.2.2

a ↑ ↓ v

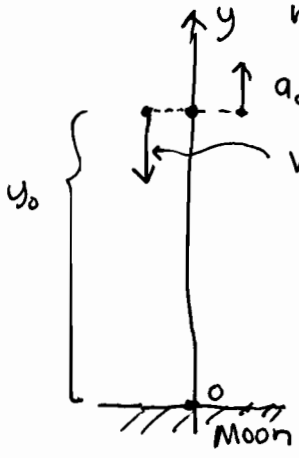
A lunar lander is freely falling toward the surface of the moon at a speed of 450 m/s.

Its retrorockets, when fired, provide a constant deceleration of 2.5 m/s² (the gravitational acceleration produced by the moon is assumed to be included in the given deceleration).

At what height above the lunar surface should the retrorockets be activated to ensure a "soft touchdown" ($v = 0$ at impact)?

Solution

1) Draw a diagram, set up variables:



soft-landing condition:

when $v(t_1) = 0$, $y(t_1) = 0$.

PLUG IN FORMULA:

$$y = \frac{1}{2}\underbrace{a_0}_{2.5}t^2 + \underbrace{V_0}_{-450}t + y_0$$

$$\downarrow$$

$$y = \frac{1}{2}(2.5)t^2 - 450t + y_0$$

WHEN

$$v = \frac{dy}{dt} = (2.5)t - 450 = 0 \rightarrow t = \frac{450}{2.5} \equiv t_1$$

$v(0) = -450$ ✓

$= 180$ (sec)

(= 3 min)

$$0 = y(t_1) = \frac{1}{2}(2.5)(180)^2 - 450(180) + y_0$$

WHERE

$$y_0 = 450(180) - \frac{1}{2}(2.5)(180)^2$$

$= 40\,500$ m $= 40.5$ km (≈ 25.3 mi)

for metric challenged Americans

Final Answer: Fire the retrorockets at height 40.5 km above surface of moon.