E&P 7.4 39 2 mass-3 spring system initially at rest but with nonzero initial velocity

What happens if we allow nonzero initial velocity?

The $\vec{x}(0) = \langle 0,0\rangle$ equations are unchanged, forcing $c_1 - 5 = 0$, $c_5 - 4 = 0$ as before.

The $\vec{x}\,'(0) = \langle 0,6\rangle$ equations become

$$\begin{bmatrix} x_1'(0) \\ x_2'(0) \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} c_2 \\ 2c_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \end{bmatrix} \to \begin{bmatrix} c_2 \\ 2c_4 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 6 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix} \to \begin{array}{l} c_2 = 4 \\ 2c_4 = 2 \to c_4 = 1 \end{array}$$

Substituting into:

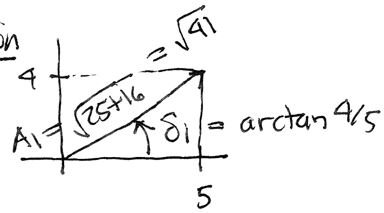
$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} c_1 \cos t + c_2 \sin t & -5\cos 3t \\ c_3 \cos 2t + c_4 \sin 2t & -4\cos 3t \end{bmatrix}$$

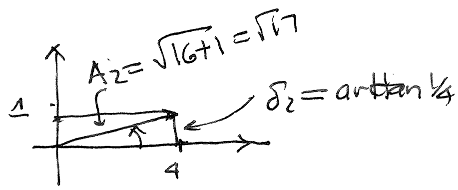
$$= \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 5\cos t \boxed{+ 4\sin t} & -5\cos 3t \\ 4\cos 2t \boxed{+ \sin 2t} & -4\cos 3t \end{bmatrix}$$

only change

$$= \underbrace{(5\cos t + 4\sin t)}_{\substack{\sqrt{41}\cos(t - \arctan 4/5) \\ \approx 6.4}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \underbrace{(4\cos 2t + \sin 2t)}_{\substack{\sqrt{17}\cos(2t - \arctan 1/4) \\ \approx 4.1}}\begin{bmatrix} -2 \\ 1 \end{bmatrix} + \cos 3t \begin{bmatrix} -3 \\ 9 \end{bmatrix}$$

Interpretation





first peaks $t = \arctan 4/5$ ≈ 0.67

$2t = \arctan 1/4 \to t = \frac{1}{2}\arctan 1/4$ ≈ 0.12

oscillation along vector length $3\sqrt{10} \approx 9.5$

comparable to natural mode oscillations

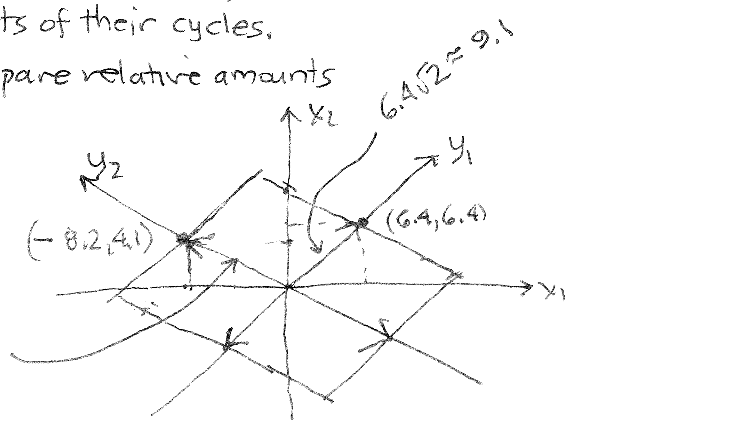
2 natural vibration modes delayed in time compared to driving force / response oscillation.

but since shifts occur at different frequencies, later peaks occur at different parts of their cycles.

amplitudes give us a way to compare relative amounts of two natural modes:

$\vec{x}_h$ is confined to this rectangle

nearly same displacements along 2 mode axes



$4.1\sqrt{5}$
≈ 9.2