15.6.8 $\quad x''+3x'+5x=-4\cos 5t \;,\; x(0)=0,\; x'(0)=0.$

$x\sim e^{rt} \to r^2+3r+5=0$

$r=\cdots=-\frac{3}{2}\pm\frac{\sqrt{11}}{2}i$

$x_h= e^{-3/2t}\left(c_1\cos\frac{\sqrt{11}}{2}t+c_2\sin\frac{\sqrt{11}}{2}t\right)$

find particular soln

$5\,[\,x_p= c_3\cos 5t+c_4\sin 5t\,]$

$3\,[\,x_p'=-5c_3\sin 5t+5c_4\cos 5t\,]$

$1\,[\,x_p''=-25c_3\cos 5t-25c_4\sin 5t\,]$

$x_p''+3x_p'+5x_p=\underbrace{[(5-25)c_3+15c_4]}_{=-4}\cos 5t+\underbrace{[-15c_3+(5-25)c_4]}_{=0}\sin 5t$

$=-4\cos 5t$

$-20c_3+15c_4=-4$

$-15c_3-20c_4=0$

$\underbrace{\begin{bmatrix}-20&15\\-15&-20\end{bmatrix}}_{5\begin{bmatrix}-4&3\\-3&-4\end{bmatrix}}\begin{bmatrix}c_3\\c_4\end{bmatrix}=\begin{bmatrix}-4\\0\end{bmatrix}\to\begin{bmatrix}c_3\\c_4\end{bmatrix}=\frac{1}{5}\;\frac{1}{\underbrace{16+9}_{25}}\begin{bmatrix}-4&-3\\3&-4\end{bmatrix}\begin{bmatrix}-4\\0\end{bmatrix}=\frac{-4}{125}\begin{bmatrix}-4\\3\end{bmatrix}=\frac{4}{125}\begin{bmatrix}4\\-3\end{bmatrix}$

$x_p=\frac{4}{125}(4\cos 5t-3\sin 5t)$

initial conditions:

$x=x_h+x_p= e^{-\frac{3}{2}t}\left(c_1\cos\frac{\sqrt{11}}{2}t+c_2\sin\frac{\sqrt{11}}{2}t\right)+\frac{4}{125}(4\cos 5t-3\sin 5t)$

$x'=-\frac{3}{2}e^{-\frac{3}{2}t}\left(c_1\cos\frac{\sqrt{11}}{2}t+c_2\sin\frac{\sqrt{11}}{2}t\right)+\frac{4}{125}(-20\sin 5t-15\cos 5t)$

$\quad+e^{-\frac{3}{2}t}\left(-\frac{\sqrt{11}}{2}c_1\sin\frac{\sqrt{11}}{2}t+\frac{\sqrt{11}}{2}c_2\cos\frac{\sqrt{11}}{2}t\right)$

$x(0)=c_1+\frac{16}{125}=0\to c_1=-\frac{16}{125}$ 8.2

$x'(0)=-\frac{3}{2}c_1+\frac{\sqrt{11}}{2}c_2-\underbrace{\frac{60}{125}}_{=\frac{12}{25}}=0\to c_2=\frac{2}{\sqrt{11}}\left(\frac{3}{2}c_1+\frac{12}{25}\right)=\frac{2}{\sqrt{11}}\left(-\frac{24}{125}+\frac{12\cdot 5}{25\cdot 5}\right)$

$=\frac{2}{\sqrt{11}}\left(\frac{36}{125}\right)=\frac{72}{125\sqrt{11}}$ 8.9

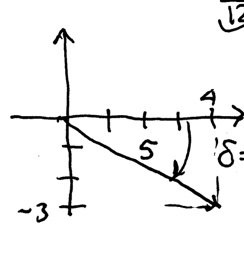
IVP soln: $x=\frac{8}{125}e^{-\frac{3}{2}t}\left(-2\cos\frac{\sqrt{11}}{2}t+\frac{9}{\sqrt{11}}\sin\frac{\sqrt{11}}{2}t\right)+\underbrace{\frac{4}{125}(4\cos 5t-3\sin 5t)}$

transient soln

$\underbrace{\frac{4}{125}(5)}_{\frac{4}{25}=A}\cos(5t+\arctan 3/4)$ steady state soln

Maple rationalizes radical:

$x_h=e^{-\frac{3}{2}t}\left(-\frac{16}{125}\cos\frac{\sqrt{11}}{2}t+\frac{72\sqrt{11}}{1375}\sin\frac{\sqrt{11}}{2}t\right)$




$\delta=-\arctan 3/4$

amplitude & phase shift of response function.