E&P 5.6 resonance calculations

(15) $x'' + 2x' + 2x = 2\cos\omega t$

$k_0 = 2,\ t_0 = \frac{1}{2},\ \omega_0 = \sqrt{2} \approx 1.414,\ Q = \frac{\sqrt{2}}{2} \approx 0.71$

$[Q = 1/\sqrt{2}$ case$]$ slightly underdamped
no resonance peak.

$2\,[x_p = c_3\cos\omega t + c_4 \sin\omega t]$

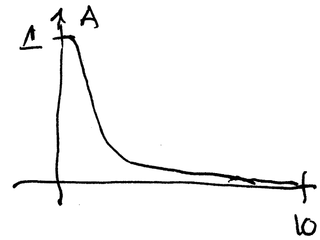
$2\,[x_p' = -c_3\omega \sin\omega t + c_4\omega\cos\omega t]$

$1\,[x_p'' = -\omega^2 c_3 \cos\omega t - \omega^2 c_4 \sin\omega t]$

$$x_p'' + 2x_p' + 2x_p = [\underbrace{(2-\omega^2)c_3 + 2\omega c_4}_{=2}]\cos\omega t + [\underbrace{-2\omega c_3 + (2-\omega^2)c_4}_{=0}]\sin\omega t = 2\cos\omega t$$

$$\begin{bmatrix} 2-\omega^2 & 2\omega \\ -2\omega & 2-\omega^2 \end{bmatrix}\begin{bmatrix} c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} c_3 \\ c_4 \end{bmatrix} = \frac{1}{(2-\omega^2)^2 + 4\omega^2}\begin{bmatrix} 2-\omega^2 & -2\omega \\ 2\omega & 2-\omega^2 \end{bmatrix}\begin{bmatrix} 2 \\ 0 \end{bmatrix} = \frac{2}{\omega^4+4}\begin{bmatrix} 2-\omega^2 \\ 2\omega \end{bmatrix}$$

$$A(\omega) = \frac{\sqrt{(2-\omega^2)^2 + 4\omega^2}}{\omega^4+4} = \frac{1}{\sqrt{\omega^4+4}}$$



(17) $x'' + 6x' + 45x = 50\cos\omega t$

$k_0 = 6,\ t_0 = \frac{1}{6} \approx 0.167,\ \omega_0 = \sqrt{45} \approx 6.71,\ Q \approx 1.12$

slightly underdamped,
small resonance peak.

$45\,[x_p = c_3\cos\omega t + c_4\sin\omega t]$

$6\,[x_p' = -\omega c_3 \sin\omega t + \omega c_4 \cos\omega t]$

$1\,[x_p'' = -\omega^2 c_3\cos\omega t - \omega^2 c_4 \sin\omega t]$

$$x_p'' + 6x_p' + 45x_p = ((45-\omega^2)c_3 + 6\omega c_4]\cos\omega t + [-6\omega c_3 + (45-\omega^2)c_4]\sin\omega t = 50\cos\omega t$$

$$\begin{bmatrix} 45-\omega^2 & 6\omega \\ -6\omega & 45-\omega^2 \end{bmatrix}\begin{bmatrix} c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 50 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} c_3 \\ c_4 \end{bmatrix} = \frac{1}{(45-\omega^2)^2 + 36\omega^2}\begin{bmatrix} 45-\omega^2 & -6\omega \\ 6\omega & 45-\omega^2 \end{bmatrix}\begin{bmatrix} 50 \\ 0 \end{bmatrix} = \frac{50}{(\dots)}\begin{bmatrix} 45-\omega^2 \\ 6\omega \end{bmatrix}$$

$$A(\omega) = \frac{50}{(\dots)}\sqrt{(45-\omega^2)^2 + 36\omega^2} = \frac{50}{\sqrt{(45-\omega^2)^2 + 36\omega^2}}$$

expand out: $\omega^4 - 54\omega^2 + 2025$