EBP 5.5.9

$y'' + 2y' - 3y = 1 + xe^x$ ($e^{0x} \to r=0, m=1$; $xe^x \to r=1, m=2$)

$\Big\} (r-0)(r-1)^2 = 0$

$D(D-1)^2(1+xe^x) = 0$

$y = e^{rx}$

$r^2 + 2r - 3 = 0$

$(r-1)(r+3) = 0$

$r = -3, 1$

$y_h = c_1 e^{-3x} + c_2 e^x$

$D(D-1)^2[(D^2+2D-3)y = 1 + xe^x]$

$D(D-1)^2(D-1)(D+3)y = 0$

$D(D-1)^3(D+3)y = 0$

$r = -3, 1, 0$
$m = 1, 3, 0$
$\Big\} \to y = \underbrace{c_1 e^{-3x}}_{y_h} + \underbrace{(c_2 + c_3 x + c_4 x^2)e^x + c_5}_{y_p}$

$-3[y_p = (c_3 x + c_4 x^2)e^x + c_5]$

$+2[y_p' = c_3 e^x + (c_3 x + c_4 x^2)e^x = [c_3 + (c_3 + 2c_4)x + c_4 x^2]e^x]$

$+1[y_p'' = [(c_3 + 2c_4) + 2c_4 x]e^x + [c_3 + (c_3 + 2c_4)x + c_4 x^2]e^x]$

$= [(2c_3 + 2c_4) + (c_3 + 4c_4)x + c_4 x^2]e^x$

$y_p'' + 2y_p' - 3y_p = \Big(-3(c_3 x + c_4 x^2) + 2[c_3 + (c_3 + 2c_4)x + c_4 x^2] + [(2c_3 + 2c_4) + (c_3 + 4c_4)x + c_4 x^2]\Big)e^x + (-3c_5)$

$= [(4c_3 + 2c_4) + [(-3+2+1)c_3 + (\quad + 4)c_4]x + (-3+2+1)c_4 x^2]e^x + (-3c_5)$

$= [(4c_3 + 2c_4) + (8c_4)x]e^x - 3c_5$

$= 1 + xe^x \longrightarrow$

$4c_3 + 2c_4 = 0$ (no $e^x$ terms on RHS)

$8c_4 = 1 \to c_4 = 1/8$

$-3c_5 = 1 \to c_5 = -1/3$

$c_3 = -\frac{1}{2}c_4 = -\frac{1}{16}$

$y_p = \left(-\frac{x}{16} + \frac{x^2}{8}\right)e^x - \frac{1}{3}$

$= \left(\frac{2x^2 - x}{16}\right)e^x - \frac{1}{3}$

I admit I had trouble keeping terms straight!

$y = y_h + y_p.$