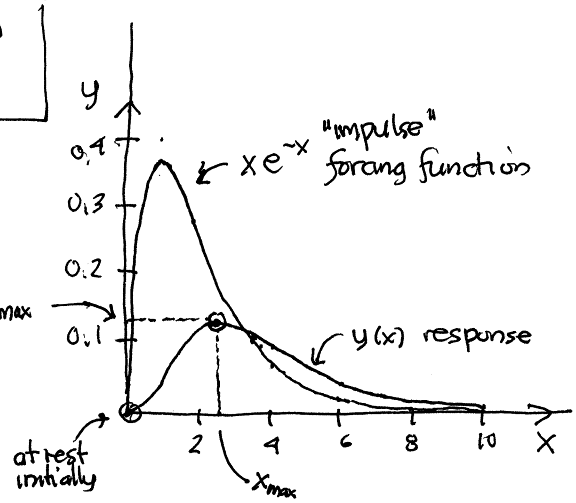
E&P 5.5.9 → altered

$$\boxed{\begin{array}{l} y''+3y'+2y = xe^{-x},\ y(0)=0,\ y'(0)=0 \\ \text{Find } |y_{max}| \text{ and } x_{max} \text{ where it occurs} \end{array}}$$



part 1 finds (see web link)

$y_h = c_1e^{-2x} + c_2e^{-x}$ (overdamped spring system if $x \sim t$ time)

$y_p = (c_3x + c_4x^2)\,e^{-x}$ so

backsub into DE:

$2\,[\ y_p = (c_3x+c_4x^2)e^{-x}$

$3\,[\ y_p' = -(c_3x+c_4x^2)e^{-x} + (c_3+2c_4x)e^{-x}$

$\quad = (c_3 + (2c_4-c_3)x - c_4x^2)\,e^{-x}$

$1\,[\ y_p'' = (-c_3-(2c_4-c_3)x+c_4x^2)\,e^{-x} + ((2c_4-c_3) - 2c_4x]\,e^{-x}$

$$y_p''+3y_p'+2y_p = \left[\begin{matrix} 2c_4x^2 \\ -3c_4x^2 \\ +c_4x^2 \end{matrix} \quad \left(\begin{matrix} 4\cancel{2c_3} \\ +6c_4-3c_3 \\ -2c_4+c_3 \quad -2c_4 \end{matrix}\right)x + \left(\begin{matrix} 3c_3 \\ -c_3 \\ +2c_4-c_3 \end{matrix}\right)\right]e^{-x}$$

$$= [\ 0 \qquad +\underbrace{2c_4}_{=1}x \qquad \underbrace{+c_3+2c_4}_{=0}\ ]\,e^{-x} = xe^{-x}$$

$c_4 = \frac{1}{2}$, $c_3 = -2c_4 = -1$ so $y_p = (\frac{1}{2}x^2 - x)\,e^{-x}$

$y = c_1e^{-2x} + c_2e^{-x} + (\frac{1}{2}x^2-x)e^{-x}$

inits: $y' = -2c_1e^{-2x} - c_2e^{-x} + (x-1)e^{-x} - (\frac{1}{2}x^2-x)e^{-x}$

$y(0) = c_1+c_2 \quad = 0$

$y'(0) = -2c_1-c_2-1 \quad = 0$

$$\begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \frac{1}{-1+2}\begin{bmatrix} -1 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

IVP soln $\boxed{y = -e^{-2x} + (1-x+\frac{1}{2}x^2)e^{-x}}$

extremize $y' = 2e^{-2x} + (-1+x)e^{-x} - (1-x+\frac{1}{2}x^2)e^{-x}$

$= [\,2e^{-2x} + (-2+2x-\frac{1}{2}x^2)e^{-x} = 0\ ]\,e^{x}$

$2e^{-x} + (-2+2x-\frac{1}{2}x^2) = 0$

$\underbrace{\frac{1}{2}x^2-2x+2}_{\frac{1}{2}(x-2)^2} = 2e^{-x}$

find intersection point

$x_{max} \approx 2.55693 \sim 2.56$

$y_{max} \approx 0.126741 \sim 0.127$

(avoid critical pt at x=0, not max)