

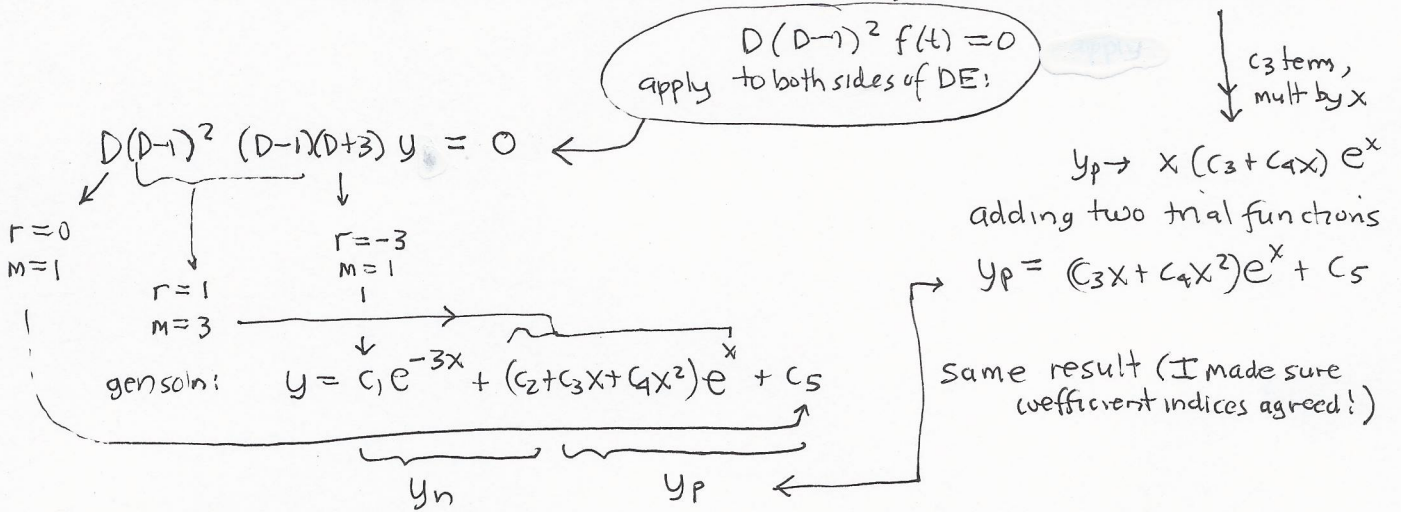
E2P3 5.5.9 Find y_p :

$y = e^{rx} \rightarrow y'' + 2y' - 3y = 1 + xe^x = f(t)$
 $r^2 + 2r - 3 = (r-1)(r+3) = 0$
 hom DE: $(D-1)(D+3)y_h = 0$
 $y_h = c_1 e^{-3x} + c_2 e^x$
 $r = 1, -3$
 $m = 1, 1$
 root overlap with RHS

$r = 1$
 $m = 2$
 $(D-1)^2(xe^x) = 0$
 gen soln:
 $y_p = (c_3 + c_4 x) e^x$
 (this term alone)

$r = 0$
 $e^{rx} = 1$
 $(D-0)1 = 0$
 gen soln:
 $y_p = c_5$
 (this term alone)

"If any term in gen soln satisfies hom DE, multiply by x , repeat if any term still satisfies hom DE..."



determine undetermined parameters:

$-3 [y_p = (c_3 x + c_4 x^2) e^x + c_5]$
 $2 [y_p' = (c_3 + 2c_4 x) e^x + (c_3 x + c_4 x^2) e^x + 0 = (c_3 + (c_3 + 2c_4)x + c_4 x^2) e^x]$
 $1 [y_p'' = (c_3 + 2c_4 + 2c_4 x) e^x + (c_3 + (c_3 + 2c_4)x + c_4 x^2) e^x]$
 $= [(2c_3 + 2c_4) + (c_3 + 4c_4)x + c_4 x^2] e^x$

$y_p'' + 2y_p' - 3y_p = \begin{matrix} (-3c_3 x - 3c_4 x^2) e^x & -3c_5 \\ + (2c_3 + 2(c_3 + 2c_4)x + 2c_4 x^2) e^x & + 0 \\ + (2c_3 + 2c_4 + (c_3 + 4c_4)x + c_4 x^2) e^x & + 0 \end{matrix}$

$= [\underbrace{(4c_3 + 2c_4)}_{=0} + \underbrace{(0c_3 + 8c_4)}_1 x + 0x^2] e^x \quad -3c_5 = 1 + x e^x$

\downarrow \downarrow \downarrow
 $c_3 = -\frac{1}{2} c_4 = -\frac{1}{16}$ $c_4 = \frac{1}{8}$ $c_5 = -\frac{1}{3}$

$y_p = \left(-\frac{1}{16} + \frac{x}{8}\right) e^x - \frac{1}{3}$