

EEP3 2.3.22. $v \leq 0$ case air resistance ($v \rightarrow v_T$)

$$\frac{dv}{dt} = -g + \rho v^2 = -g \left(1 - \frac{\rho}{g} v^2\right) = -g \left(1 - \underbrace{\left(\frac{v}{v_T}\right)^2}_{u^2}\right), \quad v_T = \sqrt{\frac{g}{\rho}}, \quad u = \frac{v}{v_T}$$

$$v_T \frac{d(v/v_T)}{dt}$$

$$\int \frac{du}{1-u^2} = \int -\frac{g}{v_T} dt = -\sqrt{pg} t + C_1 \rightarrow u = \tanh(C_1 - \sqrt{pg} t)$$

$\tanh^{-1} u$ if $|u| < 1$

$$v = v_T \tanh(C_1 - \sqrt{pg} t)$$

must be dimensionless

Given: $g = 32 \text{ ft/s}^2$
 $\rho = 0.075$ } $v_T = \sqrt{\frac{32}{0.075}} = 20.6559 \approx 20.7 \text{ ft/s}^2$

$$\sqrt{pg} = 1.54919 \text{ s}^{-1}$$

$$\tau = \frac{1}{\sqrt{pg}} = 0.645497 \text{ s}$$

I.C. $v(0) = 0$ at jump, $y(0) = 10,000 \text{ ft}$.

Since \sqrt{pg} must have inverse time units, its reciprocal must have time units — this defines the characteristic time!

$$0 = v_T \tanh C_1 \rightarrow C_1 = 0$$

$$v = v_T \tanh(-\sqrt{pg} t) = -v_T \tanh(\sqrt{pg} t)$$

$$y = \int v dt = -v_T \int \tanh \sqrt{pg} t dt$$

$$\int \tanh(ax) dx = \frac{1}{a} \ln \cosh(ax) + C$$

$$= -\frac{v_T}{\sqrt{pg}} \ln \cosh(\sqrt{pg} t) + C$$

$$10,000 = y(0) = -\frac{v_T}{\sqrt{pg}} \ln \cosh(0) + C \rightarrow C = 10,000$$

$$y = -\frac{v_T}{\sqrt{pg}} \ln \cosh(\sqrt{pg} t) + 10000$$

$$\frac{v_T}{\sqrt{pg}} = 13.3333 \text{ ft} = v_T \tau$$

Hits ground at $y=0$, solve for t : (use $\sqrt{pg} = \frac{1}{\tau}$)

$$0 = -v_T \tau \ln \cosh(t/\tau) + 10000$$

$$\ln \cosh(t/\tau) = \frac{10000}{v_T \tau}$$

$$\cosh(t/\tau) = e^{10000/(v_T \tau)}$$

$$\frac{t}{\tau} = \cosh^{-1} e^{10000/(v_T \tau)}$$

$$t = \tau \cosh^{-1} e^{10,000/(v_T \tau)}$$

↑ numerical factor ≈ 750.255
 sets scale of result!

distance traveled at terminal velocity for characteristic time interval (sets scale for change in y)

number of multiples of this distance in initial height 10,000 ft: 750

← nearly the same!

$$= 484.570 \approx 480 + 5 = \boxed{8 \text{ min } 5 \text{ s}}$$