MAT2705-02/06 10F Snow Day HW due Friday. Print Name (Last, First) _____|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1.5.37 A 400-gal tank initially contains 100 gal of brine containing 50 lb of salt. Brine containing 1 lb of salt per gallon enters the tank at the rate of 5 gal/s, and the well-mixed brine in the tank flows out at the rate of 3 gal/s. How much salt will the tank contain when it is full of brine? What are the initial and final concentrations in the tank, compared to the incoming concentration?

Start by identifying all the parameters that enter into the IVP: $\frac{dx}{dt} = r_i c_i - \frac{r_o}{V_0 + (r_i - r_o)t} x$, $x(0) = x_0$.

Incoming and outgoing flow rates r_i, r_o, incoming concentration c_i, initial volume V_0, and initial amount of solute x_0. At the end make a rough diagram that describes the situation, clearly labeling all details.

▶ solution

"identify" means state all values

$V_0 = 100$ (gal)

$x_0 = 50$ (lb)

$c_i = 1$ (lb/gal)

$r_i = 5$ (gal/s)

$r_o = 3$ (gal/s)

$V_{max} = 400$ (gal)

$$\frac{dx}{dt} = 5(1) - \frac{3}{100+(5-3)t} x$$

$$= 5 - \frac{3}{100+2t} x$$

$$\boxed{\frac{dx}{dt} + \frac{3}{100+2t} x = 5, \quad x(0) = 50}$$ standard 1st order linear IVP

$$e^{\int \frac{3}{100+2t} dt} = e^{\int \frac{3}{u}\frac{du}{2}} = e^{\frac{3}{2}\ln u} = (100+2t)^{3/2}$$

$$\frac{d}{dt}\left(x(100+2t)^{3/2}\right) = 5(100+2t)^{3/2}$$

$$x(100+2t)^{3/2} = 5\int (100+2t)^{3/2} dt = 5\int u^{3/2}\frac{du}{2}$$

$$= \frac{5}{2}\frac{(100+2t)^{5/2}}{5/2} + C = (100+2t)^{5/2} + C$$

$$x = \frac{(100+2t)^{5/2}}{(100+2t)^{3/2}} + \frac{C}{(100+2t)^{3/2}} = 100 + 2t + \frac{C}{(100+2t)^{3/2}}$$

$$50 = x(0) = 100 + \frac{C}{(100)^{3/2}} \rightarrow C = -50(10^3) = -50{,}000$$

$$\boxed{x = 100 + 2t - \frac{50\,000}{(100+2t)^{3/2}}}$$ agrees with Maple!

$$V = 100 + 2t = 400 \rightarrow t = \frac{400-100}{2} = 150 \text{ (s)}$$

$$x(150) = \underbrace{100+300}_{=400} - \frac{50{,}000}{(400)^{3/2}}$$

$$= 400 - \frac{5\times 10^4}{8\times 10^3} = 400 - 6.25$$

$$= \boxed{393.75 \text{ lb}}$$ agrees with back of book!

(amount of salt when tank overflows)

$$c(t) = \frac{x(t)}{V(t)} \qquad c_i = 1 \text{ lb/gal}$$

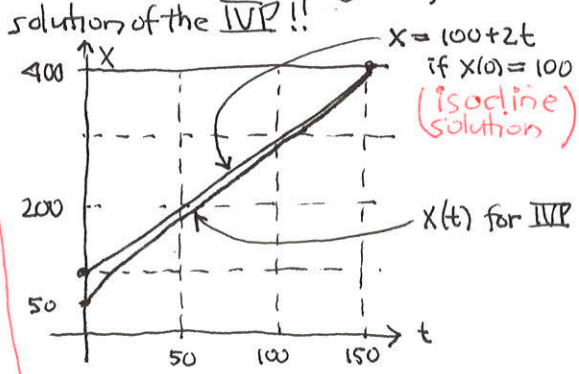
$$c(0) = \frac{\cancel{393.75}\ 50}{100} = 0.5 \text{ lb/gal}$$

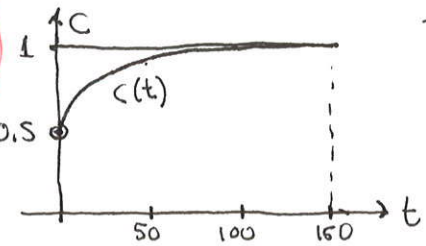
$$c(150) = \frac{393.75}{400} = 0.984 \text{ lb/gal}$$

The initial concentration has risen to nearly the incoming concentration, as one might expect.

"Diagram" in a math class probably means a math plot!!

We don't need to make a tank diagram. This does not tell us anything about the solution of the IVP!!





optional