earthquake radical blues

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}'' = \underbrace{\begin{bmatrix} -20 & 10 \\ 10 & -10 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \overbrace{E\omega^2 \cos\omega t \underbrace{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}_{\vec b}}^{\vec f}$$

earthquake: $E = 3$ inches $\to x_1, x_2$ inches

$T = \frac{2\pi}{\omega} = 3\,\text{sec} \to \omega = \frac{2\pi}{3} \approx 2.094$ ($\rightsquigarrow$ 1.954 close!)

$$|A-\lambda I| = \begin{vmatrix} -20-\lambda & 10 \\ 10 & -10-\lambda \end{vmatrix} = (\lambda+20)(\lambda+10) - 100 = \lambda^2 + 30\lambda + 100 = 0$$

$$\lambda_\pm = \frac{-30 \pm \sqrt{30^2 - 400}}{2} = -15 \pm 5\sqrt5 \approx -3.820, -26.180$$

$\to \omega_\mp = \sqrt{-\lambda_\mp} \approx 5.117, 1.954$ resonant frequencies

$T_\mp = 2\pi/\omega_\mp \approx 1.228, 3.215$ resonant periods (sec)

$$A - \lambda_\pm I = \begin{bmatrix} -20+15\mp5\sqrt5 & 10 \\ 10 & -10+15\mp5\sqrt5 \end{bmatrix} \to \begin{bmatrix} 10 & 5\mp5\sqrt5 \\ 0 & 0 \end{bmatrix}$$

$$\to \begin{bmatrix} \overset{L}{1} & \overset{F}{\tfrac12(1\mp\sqrt5)} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \to \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = t \underbrace{\begin{bmatrix} -\tfrac12(1\mp\sqrt5) \\ 1 \end{bmatrix}}_{\vec b_\pm}$$

$$B = \langle \vec b_+ | \vec b_- \rangle = \begin{bmatrix} -\tfrac12(1-\sqrt5) & -\tfrac12(1+\sqrt5) \\ 1 & 1 \end{bmatrix}$$

$$B^{-1} = \frac{1}{\sqrt5} \begin{bmatrix} 1 & \tfrac12(1+\sqrt5) \\ -1 & -\tfrac12(1-\sqrt5) \end{bmatrix} \qquad B^{-1}\vec b = \frac{1}{\sqrt5} \begin{bmatrix} \tfrac12(3+\sqrt5) \\ \tfrac12(-3+\sqrt5) \end{bmatrix}$$

$$A_B = \begin{bmatrix} -15+5\sqrt5 & 0 \\ 0 & -15-5\sqrt5 \end{bmatrix}$$

$$\vec y\,'' = A_B \vec y + B^{-1}\vec f$$

$$y_1'' = -\omega_+^2 y_1 + E\omega^2 \cos\omega t \, \frac{1}{2\sqrt5}(3+\sqrt5)$$

$$y_2'' = -\omega_-^2 y_2 + E\omega^2 \cos\omega t \, \frac{1}{2\sqrt5}(-3+\sqrt5)$$

$$y_1 = c_1 \cos\omega_+ t + c_2 \sin\omega_+ t + c_5 \cos\omega t$$

$$y_2 = \underbrace{c_3 \cos\omega_- t + c_4 \sin\omega_- t}_{y_{ih}} + \underbrace{c_6 \cos\omega t}_{y_{ip}}$$

If backsubstitute y_{ip} get:

$$(-\omega^2 + \omega_+^2) c_5 = \frac{E\omega^2}{2\sqrt5}(3+\sqrt5) \to c_5 = \frac{E\omega^2(3+\sqrt5)}{2\sqrt5[5(3-\sqrt5)-\omega^2]}$$

$$(-\omega^2 + \omega_-^2) c_6 = \frac{E\omega^2}{2\sqrt5}(-3+\sqrt5) \to c_6 = \frac{E\omega^2(-3+\sqrt5)}{2\sqrt5[5(3+\sqrt5)-\omega^2]}$$

but then $\vec x_p = B^{-1}\begin{bmatrix} c_5 \\ c_6 \end{bmatrix} \cos\omega t$ — tedious radical algebra (too much even for drbob) to get x_1-x_2 components of this vector: $\langle \alpha, \beta \rangle$

Instead use trial functions directly in x_1-x_2 equations:

$$\begin{bmatrix} x_{1p} \\ x_{2p} \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \cos\omega t \text{ (backsub:) } -\omega^2 \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \cos\omega t = \begin{bmatrix} -20 & 10 \\ 10 & -10 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \cos\omega t + E\omega^2 \cos\omega t \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

so
$$\begin{bmatrix} \omega^2-20 & 10 \\ 10 & \omega^2-10 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = -E\omega^2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} \to \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \frac{-E\omega^2}{\omega^4 - 30\omega^2 + 100} \begin{bmatrix} \omega^2-10 & -10 \\ -10 & \omega^2-20 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{-E\omega^2}{(\omega^2-\omega_+^2)(\omega^2-\omega_-^2)} \begin{bmatrix} \omega^2-20 \\ \omega^2-30 \end{bmatrix}$$

compare:
$$\lambda^2 + 30\lambda + 100 = (\lambda - \lambda_+)(\lambda - \lambda_-) = (\lambda + \omega_+^2)(\lambda - \omega_-^2)$$
$$\text{so } (-\omega^2)^2 + 30(-\omega^2) + 100 = (-\omega^2 + \omega_+^2)(-\omega^2 + \omega_-^2)$$

$$\begin{bmatrix} x_{1p} \\ x_{2p} \end{bmatrix} = \frac{-E\omega^2 \cos\omega t}{(\omega^2-\omega_+^2)(\omega^2-\omega_-^2)} \begin{bmatrix} \omega^2-20 \\ \omega^2-30 \end{bmatrix} \xrightarrow[\omega \sim 3\text{ sec period}]{E = 3\text{ in}} \begin{bmatrix} -16.63 \\ -27.28 \end{bmatrix} \cos\omega t$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = (c_1 \cos\omega_+ t + c_2 \sin\omega_+ t)\vec b_+ + (c_3 \cos\omega_- t + c_4 \sin\omega_- t)\vec b_- + \vec x_p$$

1.2 sec accordian mode; 3.2 sec tandem mode $\leftrightarrow$ (almost same frequency) 3 sec: ground oscillation response

We could impose no motion, equilibrium initial conditions at $t=0$ when earthquake starts $\vec x(0) = \vec 0 = \vec x\,'(0)$ but it really does not make sense to do it BY HAND.

Maple:
$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0.172 \\ -0.106 \end{bmatrix} \cos(5.12t)}_{\text{negligible in comparison}} + \underbrace{\begin{bmatrix} 16.96 \\ 27.24 \end{bmatrix} \cos(1.95t) + \begin{bmatrix} -16.63 \\ -27.28 \end{bmatrix} \cos(2.10t)}_{\text{beating: almost same frequency, amplitude}}$$

Beat frequency: $\frac12(2.10 - 1.95)$

Period: $\frac{2\pi}{.075} \approx 45$ sec