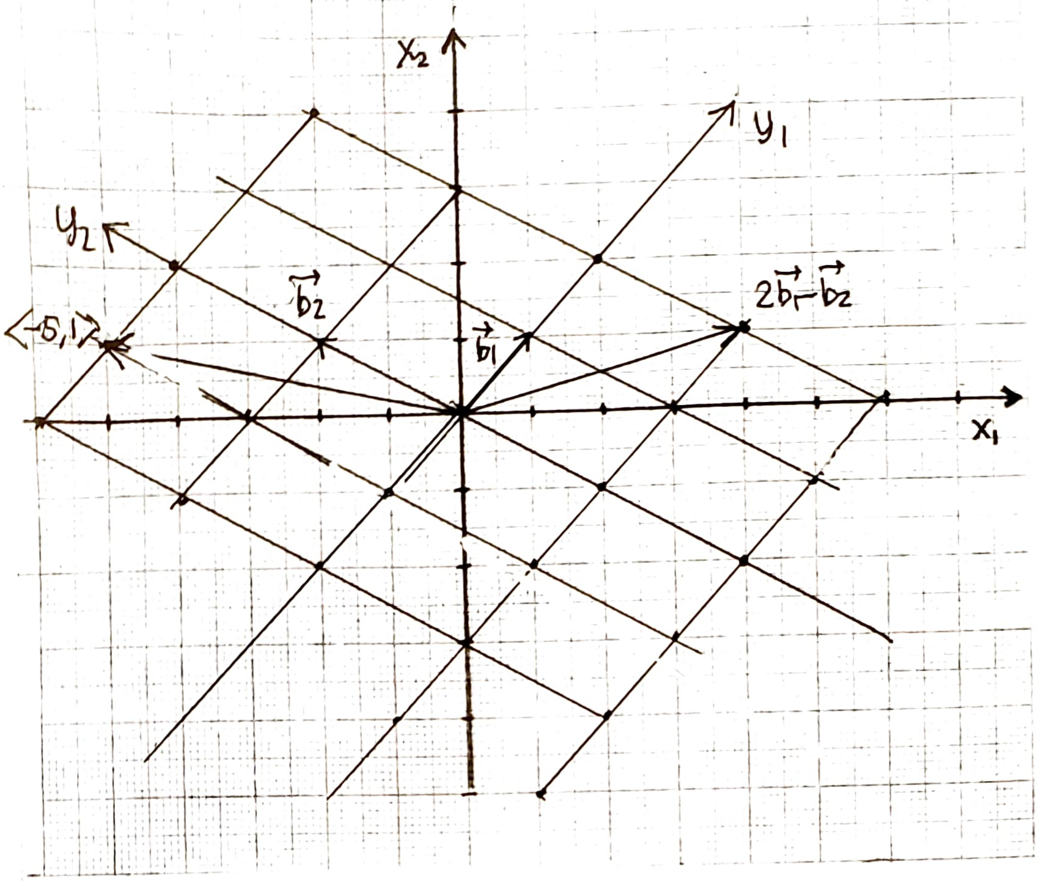
Lastname, Firstname:

worksheet 1



old basis:

$\vec{e}_1 = \langle 1,0\rangle \quad \vec{e}_2 = \langle 0,1\rangle$

new basis:

$\vec{b}_1 = \langle 1,1\rangle, \vec{b}_2 = \langle -2,1\rangle$

$$B = \langle \vec{b}_1 | \vec{b}_2 \rangle = \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}$$

basis changing matrix (columns are new basis vectors)

its inverse:

$$B^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$$

linear change of coordinates

$\vec{x}$ in terms of $\vec{y}$: $\vec{x} = B\vec{y}$

$\vec{y}$ in terms of $\vec{x}$: $\vec{y} = B^{-1}\vec{x}$

matrix form: $$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1/3 & 2/3 \\ -1/3 & 1/3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

scalar form: $x_1 = y_1 - 2y_2$

$x_2 = y_1 + y_2$

$y_1 = \frac{1}{3}(x_1 + 2x_2)$

$y_2 = \frac{1}{3}(-x_1 + x_2)$

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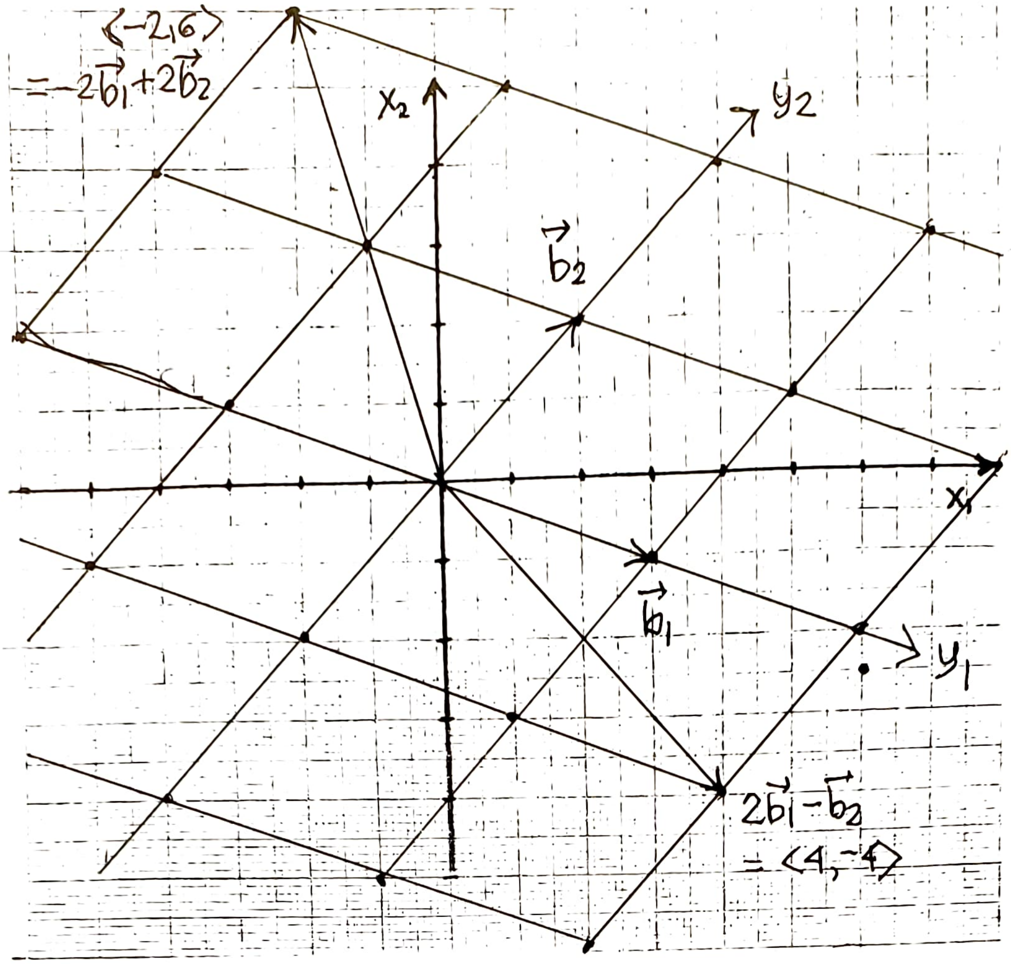
point 1: $\langle x_1, x_2\rangle = \langle -5, 1\rangle \rightarrow \langle y_1, y_2\rangle = \frac{1}{3}\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} -5 \\ 1 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} -5+2 \\ 5+1 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

$= -\vec{b}_1 + 2\vec{b}_2$

point 2: $\langle y_1, y_2\rangle = \langle 2, -1\rangle \rightarrow \langle x_1, x_2\rangle = \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 2+2 \\ 2-1 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

$= \langle 4, 1\rangle$

Instructions: Following the example handout, fill in the blanks above. Then reproduce the example graphic with $\vec{e}_1, \vec{e}_2$ then $\vec{b}_1, \vec{b}_2$. Then use a ruler to create the new 4×4 grid of "unit parallelograms" formed with edges $\vec{b}_1, \vec{b}_2$ and their translated vectors as in the example. Draw in the y_1, y_2 axes on this grid and label them. Then graphically locate and draw in the position vectors of point 1 using the x-grid and point 2 using the y-grid. Confirm that the other coordinates you read off from the other grid agree with your computation above.



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Worksheet 2

old basis:

$\vec{e}_1 = \langle 1,0\rangle \quad \vec{e}_2 = \langle 0,1\rangle$

new basis:

$\vec{b}_1 = \langle 3,-1\rangle, \vec{b}_2 = \langle 2,2\rangle$

$$B = \langle \vec{b}_1 | \vec{b}_2 \rangle = \begin{bmatrix} 3 & 2 \\ -1 & 2 \end{bmatrix}$$

basis changing matrix (columns are new basis vectors)

its inverse:

$$B^{-1} = \frac{1}{8}\begin{bmatrix} 2 & -2 \\ 1 & 3 \end{bmatrix}$$

linear change of coordinates

$\vec{x}$ in terms of $\vec{y}$: $\vec{x} = B\vec{y}$ $\qquad$ $\vec{y}$ in terms of $\vec{x}$: $\vec{y} = B^{-1}\vec{x}$

matrix form: $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ $\qquad$ $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \frac{2}{8} & -\frac{2}{8} \\ \frac{1}{8} & \frac{3}{8} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

scalar form: $x_1 = 3y_1 + 2y_2$ $\qquad$ $y_1 = \frac{1}{8}(2x_1 - 2x_2)$

$x_2 = -y_1 + 2y_2$ $\qquad$ $y_2 = \frac{1}{8}(x_1 - 3x_2)$

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point 1: $\langle x_1, x_2\rangle = \langle -2, 6\rangle \rightarrow \langle y_1, y_2\rangle = \frac{1}{8}\begin{bmatrix} 2 & -2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} -2 \\ 6 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} -4-12 \\ -2+18 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$

point 2: $\langle y_1, y_2\rangle = \langle 2, -1\rangle \rightarrow \langle x_1, x_2\rangle = \begin{bmatrix} 3 & 2 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 6-2 \\ -2-2 \end{bmatrix} = \begin{bmatrix} 4 \\ -4 \end{bmatrix}$

Instructions: Following the example handout, fill in the blanks above. Then reproduce the example graphic with $\vec{e}_1, \vec{e}_2$ then $\vec{b}_1, \vec{b}_2$. Then use a ruler to create the new 4x4 grid of "unit parallelograms" formed with edges $\vec{b}_1, \vec{b}_2$ and their translated vectors as in the example. Draw in the y_1, y_2 axes on this grid and label them. Then graphically locate and draw in the position vectors of point 1 using the x-grid and point 2 using the y-grid. Confirm that the other coordinates you read off from the other grid agree with your computation above.