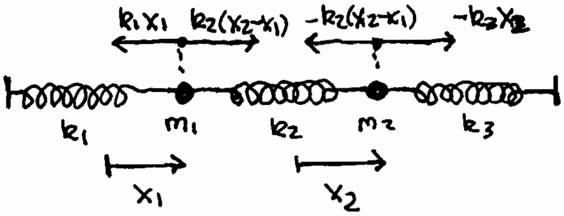
E&P3: 7.4 3,9 2 mass - 3 spring system exercize (with damping)



$$\begin{bmatrix} m_1 x_1'' \\ m_2 x_2'' \end{bmatrix} = \begin{bmatrix} -(k_1+k_2) & k_2 \\ k_2 & -(k_2+k_3) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + B_0 \cos\omega t \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \begin{bmatrix} e_1 & 0 \\ 0 & e_2 \end{bmatrix} \begin{bmatrix} x_1' \\ x_2' \end{bmatrix}$$

$$\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} x_1'(0) \\ x_2'(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

$m_1 = 1$, $m_2 = 2$
$k_1 = 1$, $k_2 = 2 = k_3$
$B_0 = 120$, $\omega = 3$
$e_1 = 1$, $e_2 = 2$

a) Plug in values, put into standard form: $\vec{x}'' = A\vec{x} + \vec{f} - I\vec{x}'$

b) Find eigenvalues λ_1, λ_2 of A: $|\lambda_1| < |\lambda_2|$ and eigenvectors $B = \langle \vec{b}_1 | \vec{b}_2 \rangle$ and $A_B = B^{-1}AB$.

c) Transform to new variables $\vec{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ by $\vec{x} = B\vec{y}$: $\vec{y}'' = A_B\vec{y} + B^{-1}\vec{f} - I\vec{y}'$

d) Solve decoupled DEs for y_1, y_2: $\vec{y} = \vec{y}_h + \vec{y}_p$.

e) Solve initial conditions for $\vec{x} = B\vec{y}$.

f) Express the result as $\vec{x} = \underbrace{y_{1h}\vec{b}_1}_{\text{mode 1}} + \underbrace{y_{2h}\vec{b}_2}_{\text{mode 2}} + \underbrace{B\vec{y}_p}_{\text{response mode}}$

Identify				
frequencies:	ω_1	ω_2	$\omega_3 = 3$	
periods:	T_1	T_2	T_3	(common period of 2π)
amplitude ratios:				

include sign → $\left(\dfrac{x_1}{x_2}\right)$

g) Return to $\vec{y}_p$ calculation and redo for a general frequency ω not equal to ω_1, ω_2 by letting $3 \to \omega$, $9 \to \omega^2$ in those calculations.

Recombine: $\vec{x}_p = B\vec{y}_p = \begin{bmatrix} \overset{A_1(\omega)}{\cancel{a(\omega)}}\cos(\omega t - \delta_1) \\ \underset{A_2(\omega)}{\cancel{b(\omega)}}\cos(\omega t - \delta_2) \end{bmatrix}$ (now distinct phase shifts)

Plot $\overset{A_1(\omega)}{\cancel{|a(\omega)|}}$, $\overset{A_2(\omega)}{\cancel{|b(\omega)|}}$ amplitudes of response functions.

h) Redo using reduction of order for $\langle x_1, x_2, x_3, x_4 \rangle = \langle x_1, x_2, x_1', x_2' \rangle$. Find 4×4 matrix eigenvalues and eigenvectors for zero damping case and for damping case.