2nd order linear homogeneous DE system : resonance

$$\underline{X}'' = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix} \underline{X} + \begin{bmatrix} 0 \\ \cos \omega t \end{bmatrix}$$

now repeat for a general frequency $\omega \neq 1, \sqrt{6}$

decoupled equations:

$$\underline{y}'' = \begin{bmatrix} -1 & 0 \\ 0 & -6 \end{bmatrix} \underline{y} + \begin{bmatrix} \frac{2}{5}\cos \omega t \\ \frac{1}{5} \cos \omega t \end{bmatrix}$$

$$y_1'' + y_1 = \frac{2}{5} \cos \omega t$$
$$y_2'' + 6y_2 = \frac{1}{5} \cos \omega t$$

the homogeneous soln doesn't change and is determined by initial conditions

we are only interested in the particular soln which is the response to the driving force.
Because there is no damping, we only need cosine functions in our trial particular functions:

$y_{1p} = c_5 \cos \omega t$

$$y_{1p}'' + y_1 = c_5(-\omega^2+1) \cos \omega t = \frac{2}{5} \cos \omega t \rightarrow c_5 = \frac{2}{5(1-\omega^2)}, \quad \omega \neq 1$$

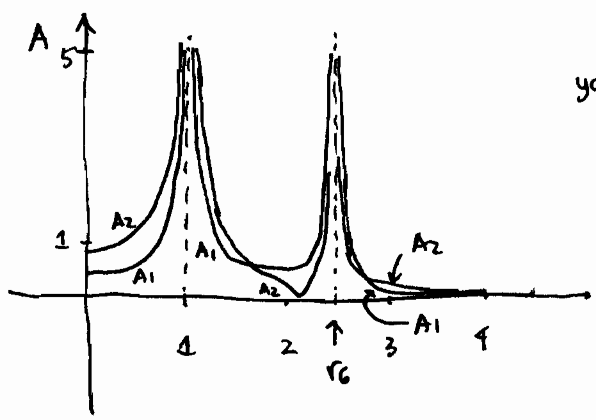
$y_{2p} = c_7 \cos \omega t$

$$y_{2p}'' + 6 y_{2p} = c_7 (-\omega^2 + 6) \cos \omega t = \frac{1}{5} \cos \omega t \rightarrow c_7 = \frac{1}{5(6-\omega^2)}, \quad \omega \neq \sqrt{6}$$

$$\begin{bmatrix} x_{1p} \\ x_{2p} \end{bmatrix} = B \begin{bmatrix} y_{1p} \\ y_{2p} \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} \frac{2}{5(1-\omega^2)} \\ \frac{1}{5(6-\omega^2)} \end{bmatrix} \cos \omega t = \begin{bmatrix} \frac{2}{5}\left(\frac{1}{1-\omega^2} - \frac{1}{6-\omega^2}\right) \\ \frac{1}{5}\left(\frac{4}{1-\omega^2} + \frac{1}{6-\omega^2}\right) \end{bmatrix} \cos \omega t = \frac{\cos \omega t}{(1-\omega^2)(6-\omega^2)} \begin{bmatrix} 2 \\ 5-\omega^2 \end{bmatrix}$$

$$= \begin{bmatrix} \pm A_1 \cos \omega t \\ \pm A_2 \cos \omega t \end{bmatrix}$$

$$A_1 = \frac{2}{|(1-\omega^2)(6-\omega^2)|}, \quad A_2 = \frac{5-\omega^2}{|(1-\omega^2)(6-\omega^2)|}$$



you need to see this in a technology plot!

If some small amount of damping were added to the system these would become resonance peaks near the natural frequencies.