MAT2705-04/05 23S Final Exam Print Name (Last, First) ____________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use equal signs and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). **You may use technology for row reductions, determinants and matrix inverses without showing details (identify technology).** Otherwise only use technology to CHECK hand calculations, not subsitute for them, unless specifically requested. Remember "exact" means no decimals!

pledge **[sign and date the pledge at the end of your exam]**

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet on top of your answer sheets as a cover page:
"During this examination, all work has been my own. I have not accessed any of the class web pages or any other sites during the exam. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date:

Shortened version for practice. Modified by italic boldface.

Consider the initial value problem:

$$x_1'' = -10\,x_1 + 3\,x_2 + 30\cos(2\,t),$$
$$x_2'' = \quad 2\,x_1 - 15\,x_2 + 30\cos(2\,t)\,,$$
$$x_1(0) = 7,\ x_2(0) = 11,$$
$$x_1'(0) = -13,\ x_2'(0) = 5$$

Solve this problem following these steps. To make sure you copy down the numbers correctly in working this problem, know that the final expressions for the unknowns only involve single digit integers, although you encounter fractions along the way.

$$x_1(0) = 7,\ x_2(0) = 11,\ \mathrm{D}(x_1)(0) = -13,\ \mathrm{D}(x_2)(0) = 5 \tag{1}$$

) ***Enter this IVP into Maple and find the solution you are then going to derive by hand. Then from the coefficients of the three frequency sinusoidal functions, identify the simplist eigenvectors asssociated with each frequency.*

a) Write this system of DEs AND initial conditions in matrix form, identifying the coefficient matrix A. and the driving force F.

) ***Bypass the hand derivation and use Maple to get the eigenvalues and eigenvectors, confirming their relationship to the eigenfrequencies and eigenvectors you found by observation from the solution. Then continue...*

b) Find the eigenvalues and eigenvectors of the coefficient matrix by hand showing all steps in the matrix solution process (identifying leading and free variables, stating the reduced matrix equations, etc.) and order the eigenvalues by increasing absolute value $|\lambda_1| < |\lambda_2|$ and **choose the smallest integer component eigenvectors** in the basis changing matrix $B = \langle b_1 | b_2 \rangle$. State its inverse B^{-1}. and the diagonalized matrix $A_B = B^{-1} A\, B$ and calculate by hand $F_B = B^{-1} F$.

c) Introduce the decoupling change of variables $x = B\,y = y_1\,b_1 + y_2\,b_2$ and re-express the matrix DE in the new variables, and then write down the corresponding decoupled scalar DEs for the new variables y_1, y_2.

d) Write down the general homogenous solutions of each decoupled DE.

e) Find the particular solution of each decoupled DE using the method of undetermined coefficients.

f) Write down the general solution for the original variables in matrix or vector form (as a linear combination) or in scalar form as you wish.
g) Impose the initial conditions on this general solution and express the resulting solution in scalar form for each of the unknowns.
) ***at this point you should have derived the Maple solution.*
h) Now rewrite your final solution in the following linear combination vector form by re-expressing the homogeneous solutions y_{1h}, y_{2h} in exact phase-shifted cosine form

$$x = A_1 \cos(\omega_1 t - \delta_1)\, b_1 + A_2 \cos(\omega_2 t - \delta_2)\, b_2 + \cos(2t)\, b_3 ,$$

supporting your two amplitude-phase-shift calculations with clearly labeled triangle diagrams in the coefficient plane of each sinsoidal function indicating your angles, and where b_3 is the vector coefficient of $\cos(2t)$ that corresponds to the particular solution in response to the driving force. You may chose either range for the phase shift evaluation (***No, use the range*** $(-\pi, \pi]$) but make sure your angle is labeled correctly in your diagrams.
Forget this part:
i) On the grid provided plot the two eigenvectors and the new coordinate axes (label everything), and then the 6 vectors $\pm A_1 b_1, \pm A_2 b_2, \pm b_3$
and finally the parallelogram whose sides correspond to the lines $y_1 = \pm A_1, y_2 = \pm A_2$. [This parallelogram confines the homogeneous solution.]
Instead draw in the eigenvectors and new coordinate axes and the vector $\langle 3, 8\rangle$ and its projection parallelogram onto the new axes, labeling those vector projections by their multiples of the eigenvectors, the sum of which equals this vector.