MAT2705-04/05 22S Take Home Test 3 Print Name (Last, First) ____________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC, MathCad). **You may use technology for row reductions, matrix inverses, determinants, plotting and root finding without showing intermediate steps.** ***Print*** the requested technology plots, labeling them and **annotating them appropriately *by hand*** and attach to the end of your test. All differential equations should be solved "by hand" unless otherwise specified.

3. Consider the following DE system IVP:

$x_1' = -5x_1 - 5x_2 - 2x_3, x_2' = 6x_1 + 6x_2 + 5x_3, x_3' = -6x_1 - 6x_2 - 5x_3, \quad x_1(0) = 1, x_2(0) = 0, x_3(0) = 1$.

a) Write down the DE system and its initial condition in explicit matrix form, identifying the coefficient matrix A.

b) Evaluate the characteristic equation and determine the ordered eigenvalues $\lambda_1(real), \lambda_2 = \lambda_+, \lambda_3 = \lambda_-$ (positive imaginary eigenvalue first).

c) Solve the equations using the reduction algorithm necessary to find an eigenbasis matrix $B = \langle \vec{b}_1 | \vec{b}_2 | \vec{b}_3 \rangle$.

d) Re-express the DE system in the new variables $\vec{y} = B^{-1}\vec{x}$ and solve the resulting decoupled equations, then write out the vector form of the general solution for the original vector variable

$$\vec{x} = \sum_{i=1}^{3} y_i \vec{b}_i .$$

e) Re-express the complex eigenvalue contributions to your solution as an explicitly real pair of modes.

f) Impose the initial conditions on your general solution, using a matrix inverse to accomplish this and express the final solution in scalar form $x_1 = ..., etc.$

g) Plot the three solution curves with the three equilibrium lines versus t for $t = 0..T$ for an appropriate decay window which shows clearly their asymptotic behavior, adjusting your time interval to see the merging to the equilibrium solution. (The limit of $\vec{x}$ as t approaches infinity is a constant vector $\langle a, b, c \rangle$. the equilibrium solution.)

Label the three graphs by their variable names [**plot#3**]. Justify your initial choice of time interval.

Advice. When in doubt about how much work to show, show more. Explain using words if it helps. Think of this take-home test as an exercise in "writing intensive" technical expression. Try to impress bob as though it were material for a job interview. In a real world technical job, you need to be able to write coherent technical reports that other people can follow. Print out your plot and hand annotate it, labeling axes and key points by hand.

Check out the solution online in the archive.

Example plot:

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> plot([x1, x2, x3, a, b, c], t = 0..10, color = [red, blue, green, gray, gray, gray])
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