tetrahedron:

$x=1,\ y=2,\ z=3$

$\frac{x}{2}+\frac{y}{4}+\frac{z}{6}=1$

solve for each variable in turn:

$x=2\left(1-\frac{y}{4}-\frac{z}{6}\right)$

$y=4\left(1-\frac{x}{2}-\frac{z}{6}\right)$

$z=6\left(1-\frac{x}{2}-\frac{y}{4}\right)$

starting values for 3 partial integrations

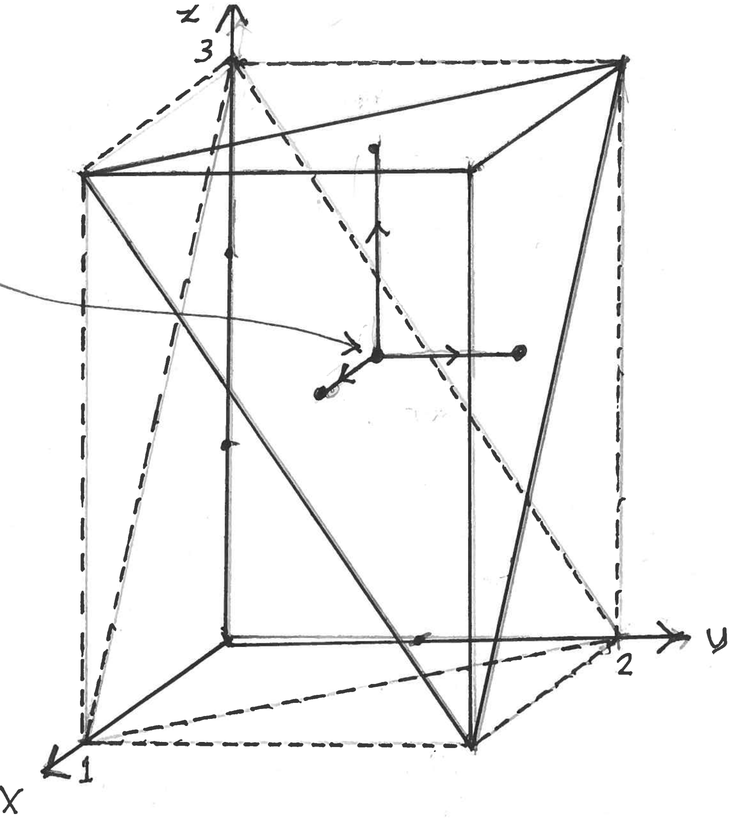
ending values:

$x=1$

$y=2$

$z=3$

these give lower & upper limits for innermost integration



6 way iteration example

innermost integral moves along coord axes from the oblique plane outward from origin

outer double integral is done over projection of solid to coordinate planes of remaining variables

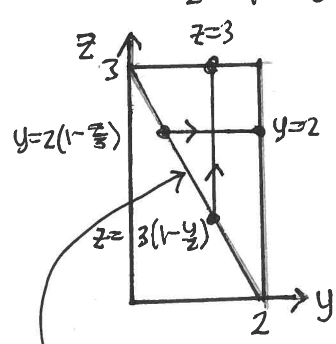
dashed triangles are those projections.

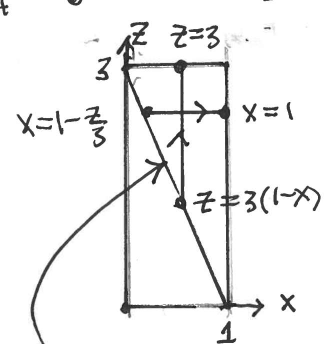
intersections of faces are common solutions of pairs of equations, eliminate one variable:

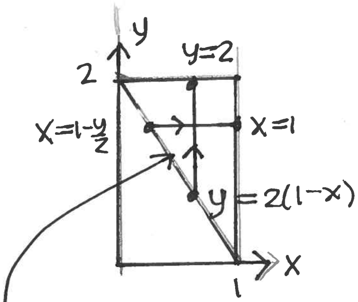
$x=1 \to \frac{x}{2}+\frac{y}{4}+\frac{z}{6}=1 \to \frac{1}{2}+\frac{y}{4}+\frac{z}{6}=1 \to \frac{y}{2}+\frac{z}{3}=1$ project solid onto yz plane

$y=2 \to \frac{x}{2}+\frac{y}{4}+\frac{z}{6}=1 \to \frac{x}{2}+\frac{2}{4}+\frac{z}{6}=1 \to x+\frac{z}{3}=1$ project solid onto xz plane

$z=3 \to \frac{x}{2}+\frac{y}{4}+\frac{z}{6}=1 \to \frac{x}{2}+\frac{y}{4}+\frac{3}{6}=1 \to x+\frac{y}{2}=1$ project solid onto xy plane







$\frac{y}{2}+\frac{z}{3}=1 \longrightarrow z=3\left(1-\frac{y}{2}\right)$, $\ y=2\left(1-\frac{z}{3}\right)$

$x+\frac{z}{3}=1 \longrightarrow z=3(1-x)$, $\ x=1-\frac{z}{3}$

$x+\frac{y}{2}=1 \longrightarrow y=2(1-x)$, $\ x=1-\frac{y}{2}$

$$\left(\int_{2\left(1-\frac{y}{4}-\frac{z}{6}\right)}^{1} f\,dx\right) \qquad \left(\int_{4\left(1-\frac{x}{2}-\frac{z}{6}\right)}^{2} f\,dy\right) \qquad \left(\int_{6\left(1-\frac{x}{2}-\frac{y}{4}\right)}^{3} f\,dz\right)$$

$$\int_0^2\int_{3\left(1-\frac{y}{2}\right)}^{3}(\ \ )\,dz\,dy \qquad \int_0^1\int_{3(1-x)}^{3}(\ \ )\,dz\,dx \qquad \int_0^1\int_{2(1-x)}^{2}(\ \ )\,dy\,dx$$

$$\int_0^3\int_{2\left(1-\frac{z}{3}\right)}^{2}(\ \ )\,dy\,dz \qquad \int_0^3\int_{1-\frac{z}{3}}^{1}(\ \ )\,dx\,dz \qquad \int_0^2\int_{1-\frac{y}{2}}^{1}(\ \ )\,dx\,dy$$

when $f(xyz)=1$, all integrals give the volume: 1.