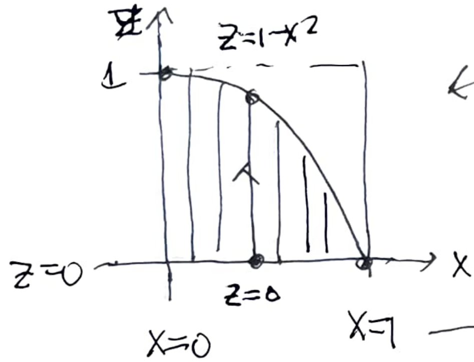
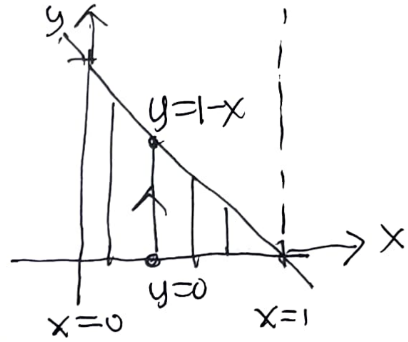
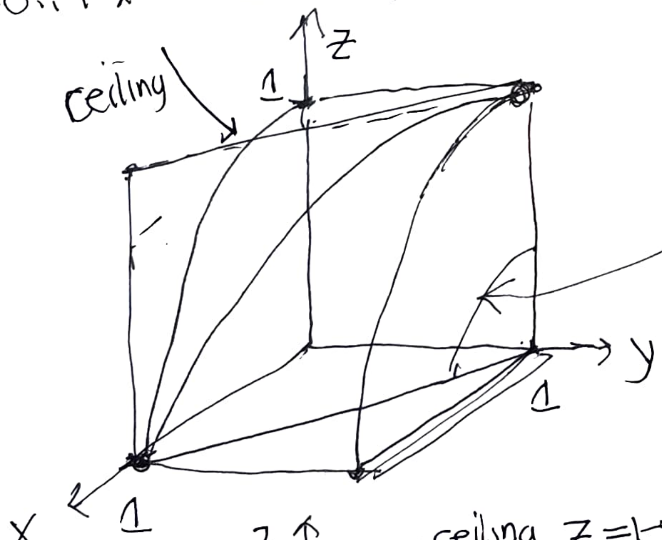
triple integral deconstruction (1)

$$Q=\int_0^1\int_0^{1-x^2}\int_0^{1-x} f\,dy\,dz\,dx \rightarrow \int_{x=0}^{x=1}\int_{z=0}^{z=1-x^2}\int_{y=0}^{y=1-x} f\,dy\,dz\,dx$$

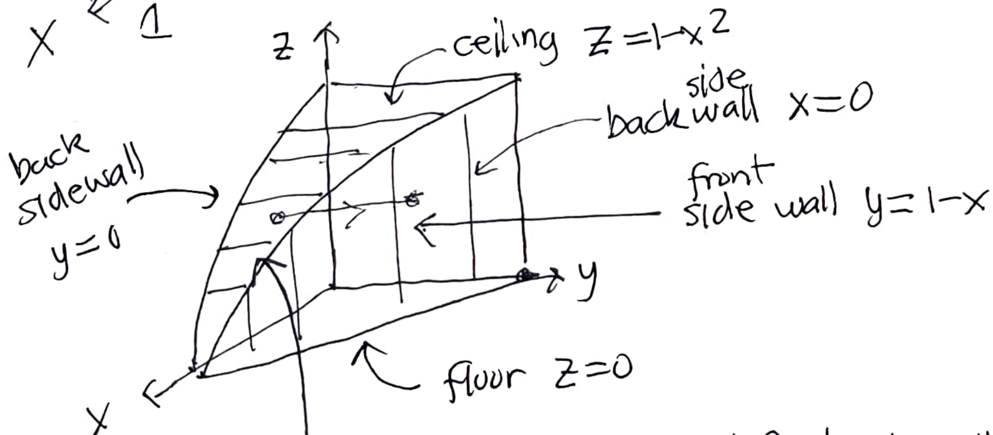




$z=0..1-x^2$ while $x=0..1$



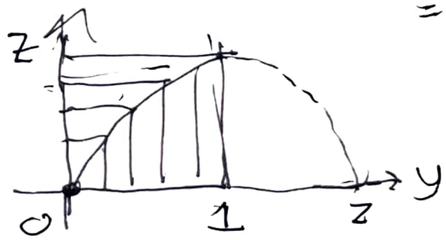
vertical side wall
must intersect ceiling
in curve above $y=x^2$
between (1,0,0) and (0,1,1)



y-first
diagram

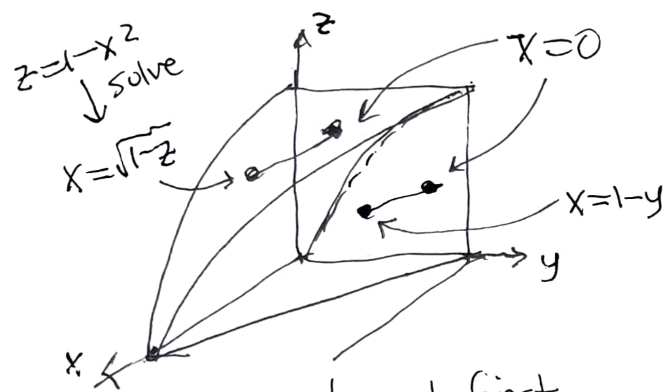
intersection of ceiling and front sidewall:

$$\left.\begin{array}{l} z=1-x^2 \\ y=1-x \end{array}\right\} \rightarrow \text{eliminate } x:\ x=1-y \rightarrow z=1-(1-y)^2$$
$$=2y-y^2=(2-y)y$$

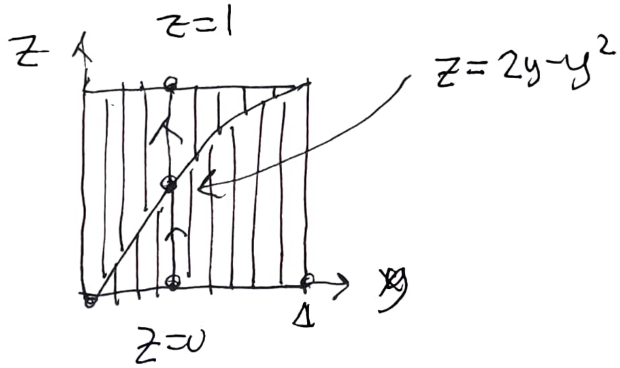


looking down x-axis
2 ceiling expressions, floor x=0
need sum of 2 triple integrals

triple integral deconstruction (2)



do x integral first
(2 separate triple integrals)

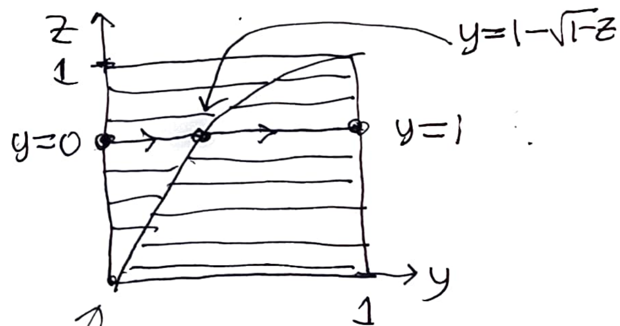


below: $x = 0 .. 1-y$
while $z = 0 .. 2y-y^2$
while $y = 0 .. 1$

top: $x = 0 .. \sqrt{1-z}$
while $z = 2y-y^2 .. 1$
while $y = 0 .. 1$

$$Q = \int_0^1 \int_0^{2y-y^2} \int_0^{1-y} f \, dx \, dz \, dy$$
$$+ \int_0^1 \int_{2y-y^2}^{1} \int_0^{\sqrt{1-z}} f \, dx \, dz \, dy$$

or



$$z = 2y - y^2 \to y^2 - 2y + z = 0$$
$$y = \frac{2 \pm \sqrt{4 - 4(1)z}}{2}$$
$$= 1 \pm \sqrt{1-z}$$

root is ≤ 1

intersection (inverted)

below (left): $x = 0 .. \sqrt{1-z}$
while $y = 0 .. 1-\sqrt{1-z}$
while $z = 0 .. 1$

above (right): $x = 0 .. 1-y$
while $y = 1-\sqrt{1-z} .. 1$
while $z = 0 .. 1$

$$Q = \int_0^1 \int_0^{1-\sqrt{1-z}} \int_0^{\sqrt{1-z}} f \, dx \, dy \, dz$$
$$+ \int_0^1 \int_{1-\sqrt{1-z}}^{1} \int_0^{1-y} f \, dx \, dz \, dy$$

exercise. do 2+1 diagrams for z-first iterations & rewrite the integral