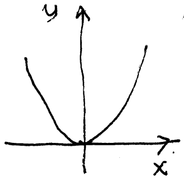
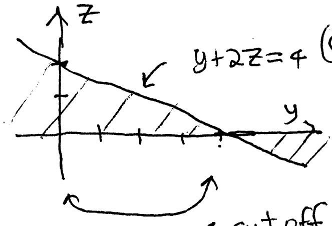
Consider the solid region enclosed by the surfaces $y=x^2$, $z=0$, $y+2z=4$.
Setup 6 different integrals representing its volume.



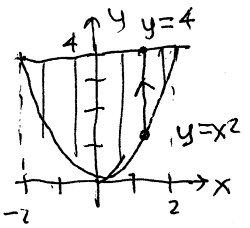
vertical side walls (extends in z direction)



$y+2z=4$ (ceiling) $\to$ $y=4-2z$, $z=(4-y)/2$

$z=0$ (floor)

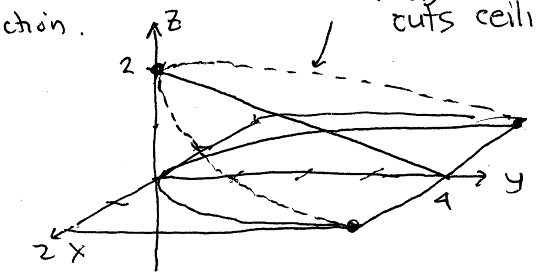
$y=0$, $y=4$ cut off solid in y-direction.



projection onto xy plane & floor surface of solid.

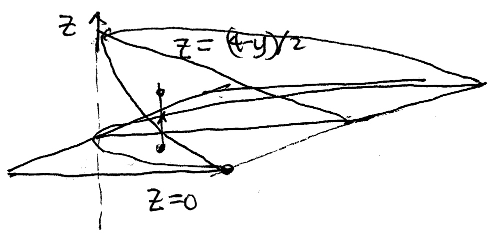
$x=-2..2$

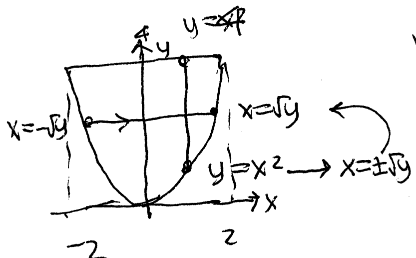
imagine where sidewall cuts ceiling plane



so solid region is this triangular wedge of the parabolic cylinder.

z-first (innermost integral)



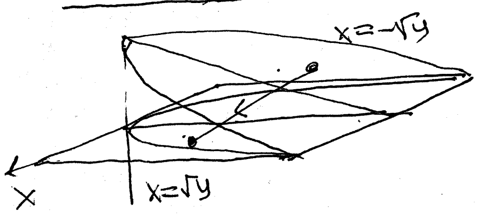


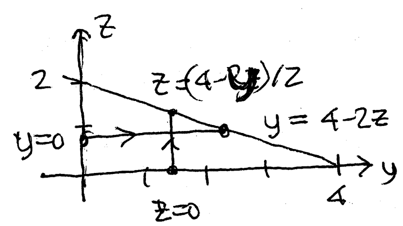
projection onto xy plane

$$V=\int_0^4\int_{-\sqrt{y}}^{\sqrt{y}}\int_0^{(4-y)/2} 1\,dz\,dx\,dy$$

$$=\int_{-2}^{2}\int_{x^2}^{4}\int_0^{(4-y)/2} 1\,dz\,dy\,dx$$

x-first (innermost integral)



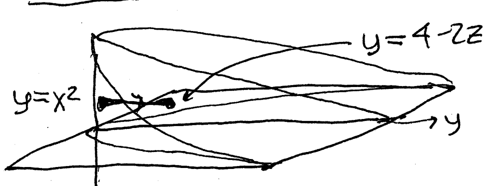


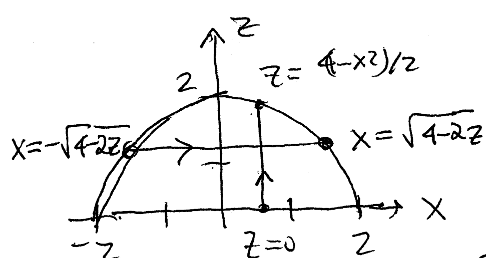
projection onto yz plane

$$V=\int_0^2\int_0^{4-2z}\int_{-\sqrt{y}}^{\sqrt{y}} 1\,dx\,dy\,dz$$

$$=\int_0^4\int_0^{(4-y)/2}\int_{-\sqrt{y}}^{\sqrt{y}} 1\,dx\,dz\,dy$$

y-first (innermost integral)





sidewall intersects ceiling?

$y=x^2$ $\quad$ $y=4-2z$

eliminate y to get xz relationship

$x^2=4-2z$ or $z=(4-x^2)/2$

$x=\pm\sqrt{4-2z}$

$$V=\int_0^2\int_{-\sqrt{4-2z}}^{\sqrt{4-2z}}\int_{x^2}^{4-2z} 1\,dy\,dx\,dz$$

$$=\int_{-2}^{2}\int_0^{(4-x^2)/2}\int_{x^2}^{4-2z} 1\,dx\,dy\,dz$$