14.R.25

$z = 3x^2 - y^2 + 2x \quad (1,-2,1)$

$\frac{\partial z}{\partial x} = 6x + 2$

$\frac{\partial z}{\partial y} = -2y$

$\vec{\nabla} z = \langle 6x+2, -2y \rangle$

$\vec{\nabla} z \big|_{x=1, y=-2} = \langle 6+2, -2(-2) \rangle = \langle 8, 4 \rangle$

$z = L(x,y) = 1 + 8(x-1) + 4(y+2)$
$= 1 + 8x - 8 + 4y + 8$

$\boxed{8x + 4y - z = -1}$

$\vec{n} = \langle 8, 4, -1 \rangle$

OR

$(1,-2,1) \quad \vec{n} = \langle 1,-2,1 \rangle$

$\rightarrow F(x,y,z) = z - 3x^2 + y^2 - 2x = 0$ (level surface)

$\vec{\nabla} F(xyz) = \langle -6x-2, 2y \rangle$

$\vec{\nabla} F(1,-2,1) = \langle -8, -4, 1 \rangle = -\vec{n}$

$0 = \vec{n} \cdot (\vec{r} - \vec{r}_0)$
$= \langle -8, -4, 1 \rangle \cdot \langle x-1, y+2, z-1 \rangle$
$= -8(x-1) - 4(y+2) + (z-1)$
$= -8x + 8 - 4y - 8 + z - 1$

$-8x - 4y + z = 1$ or

$\boxed{8x + 4y - z = -1}$

$\vec{r} = \vec{r}_0 + t\vec{n} = \langle 1,-2,1 \rangle + t\langle 8,4,-1 \rangle$

$\langle x,y,z \rangle = \langle 1+8t, -2+4t, 1-t \rangle$

variables ↑ equal symbol; way of tracing out curve in space of variables

} vector equation of normal line is a relationship between x,y,z and t, not the vector valued function of t on its RHS

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29. $\sin(xyz) = x + 2y + 3z \quad (2,-1,0)$

↓

$F(x,y,z) = \sin(xyz) - x - 2y - 3z = 0$ (level surface)

etc as above.