## numerical partial derivatives

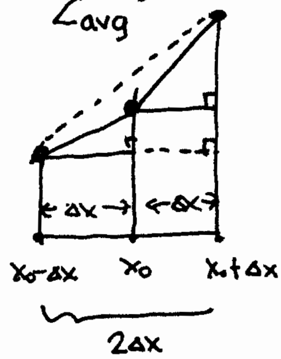
| x \ y | 1.8 | 2.0 | 2.2 |
|---|---|---|---|
| 2.5 | 12.5 | 10.2 | 9.3 |
| 3.0 | 18.1 | 17.5 | 15.9 |
| 3.5 | 20.0 | 22.4 | 26.1 |

values of $f(x,y)$

note: $\Delta x = 0.5$ constant equal
$\Delta y = 0.2$ separations

## 1d calculation of numerical derivative

$$f_x(x_0) = \underbrace{\frac{1}{2}}_{\text{avg}}\left[\underbrace{\left(\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}\right)}_{\text{right sec slope}} + \underbrace{\left(\frac{f(x_0)-f(x_0-\Delta x)}{\Delta x}\right)}_{\text{left sec slope}}\right]$$




$$= \frac{f(x_0+\Delta x)-f(x_0-\Delta x)}{2\Delta x}$$

sec slope spanning left & right adjacent data points (dotted line)

y fixed in each 1d calculation

$$f_x(3.0,1.8) = \frac{1}{2}\left[\frac{f(3.5,1.8)-f(3.0,1.8)}{\Delta x} + \frac{f(3.0,1.8)-f(2.5,1.8)}{\Delta x}\right] = \frac{1}{2(0.5)}\left[(20.0-18.1)+(18.1-12.5)\right]$$
$$= 1.9 + 5.6 = \boxed{7.5}$$

$$f_x(3.0,2.0) = \frac{1}{2}\left[\frac{f(3.5,2.0)-f(3.0,2.0)}{\Delta x} + \frac{f(3.0,2.0)-f(2.5,2.0)}{\Delta x}\right] = \frac{1}{2(0.5)}\left[(22.4-17.5)+(17.5-10.2)\right]$$
$$= 4.9 + 7.3 = \boxed{12.2}$$

$$f_x(3.0,2.2) = \frac{1}{2}\left[\frac{f(3.5,2.2)-f(3.0,2.2)}{\Delta x} + \frac{f(3.0,2.2)-f(2.5,2.2)}{\Delta x}\right] = \frac{1}{2(0.5)}\left[(26.1-15.9)+(15.9-9.3)\right]$$
$$= 10.2 + 6.6 = \boxed{16.8}$$

values of $f_x(x,y)$: (now it is a single 1d calculation of a numerical derivative)

| x \ y | 1.8 | 2.0 | 2.2 |
|---|---|---|---|
| 3.0 | 7.5 | 12.2 | 16.8 |

$$f_{xy}(3.0,2.0) = \frac{1}{2}\left[\frac{f_x(3.0,2.2)-f_x(3.0,2.0)}{\Delta y} + \frac{f_x(3.0,2.0)-f_x(3.0,1.8)}{\Delta y}\right]$$
$$= \frac{1}{2(0.4)}\left[(16.8-12.2)+(12.2-7.5)\right] = \frac{1}{.4}(4.6+4.7) = \frac{1}{.4}9.3 = \boxed{23.25}$$

pretty tedious, but worth understanding