Stewart 9e 14.3.52

$$z = \arctan\left(\frac{x+y}{1-xy}\right)$$

$$z_x = \frac{1}{1+\left(\frac{x+y}{1-xy}\right)^2} \cdot \frac{(1-xy)(1)-(x+y)(-y)}{(1-xy)^2}$$

$$= \frac{1}{\frac{(1-xy)^2+(x+y)^2}{\cancel{(1-xy)^2}}} \cdot \frac{1\underbrace{\cancel{-xy}+y^2\cancel{+xy}}_{0}}{\cancel{(1-xy)^2}_{\,1}}$$

$$= \frac{(1+y^2)}{1\underbrace{\cancel{-2xy}+x^2y^2+x^2+y^2\cancel{+2xy}}_{0}}$$

$$= \frac{1+y^2}{1+x^2y^2+x^2+y^2} \leftarrow (1+x^2)+y^2(1+x^2) = (1+y^2)(1+x^2)$$

$$= \frac{1+y^2}{(1+y^2)(1+x^2)} = \frac{1}{1+x^2}$$

$$z_{xx} = \frac{d}{dx}(1+x^2)^{-1} = -(1+x^2)^{-2}\cdot(2x) = \frac{-2x}{(1+x^2)^2}$$

$$z_{xy} = 0 \quad \leftarrow \left(= \frac{\partial}{\partial y}\left(\frac{1}{1+x^2}\right)\right)$$

$$z_{yy} = \frac{-2y}{(1+y^2)^2}$$ by symmetry. ($z$ is invariant under $x \leftrightarrow y$.)