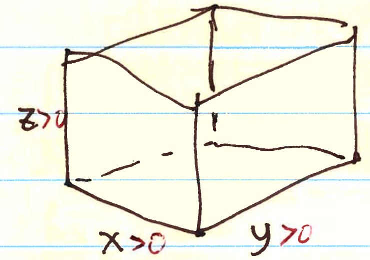
14.7 II ~~2~~ 1 word problems

A rectangular Box with no lid is to be made from $12\,m^2$ of cardboard. Find the maximum volume of the box.



3 variables BUT $x>0, y>0, z>0$

$V = xyz > 0$ (to be extremized)

$A = \underset{\text{bot (no top)}}{xy} + \underset{\text{sides}}{2xz} + \underset{\text{sides}}{2yz} = 12$ (constraint)

use to eliminate 1 variable.

$xy + 2z(x+y) = 12$

$z = \dfrac{12-xy}{2(x+y)} > 0$

$V = xy\,\dfrac{(12-xy)}{2(x+y)} = \dfrac{12xy - x^2y^2}{2(x+y)} = f(x,y)$ on $x>0$, $y>0$

note also $xy < 12$

$\dfrac{\partial V}{\partial x} = \cdots = \dfrac{y^2(12-2xy-x^2)}{2(x+y)^2} = 0$

$\dfrac{\partial V}{\partial y} = \cdots = \dfrac{x^2(12-2xy-y^2)}{2(x+y)^2} = 0$

show Maple evaluation

$x \neq 0$, $y \neq 0$ → $\boxed{12-2xy-x^2=0}$, $\boxed{12-2xy-y^2=0}$

subtract.

$y^2 - x^2 = 0$

$y = \pm x$

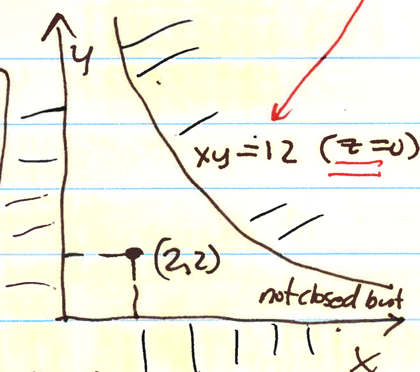
$y = x$ (since both >0)

backsub $12 - 2x^2 - x^2 = 0$

$12 = 3x^2$, $x^2 = 4$, $x = 2$ $(x>0)$

$\therefore y = 2$

$\boxed{(2,2)} \to z = \dfrac{12-4}{2(2+2)} = \dfrac{8}{8} = 1$



only missing points at ∞.

exactly 1 critical pt has to be local max. Since a largest such box clearly must exist.

(physical reasoning)

bottom dimensions should be 2m×2m, height 1m, resulting Volume = $4\,m^3$

$V = 2\cdot 2\cdot 1 = \boxed{4}$

Extremize $F(x,y,z)$ subject to $C(x,y,z) = 0$ (constraint).

solve $z = Z(x,y)$

$f(x,y) = F(x,y,z(x,y))$