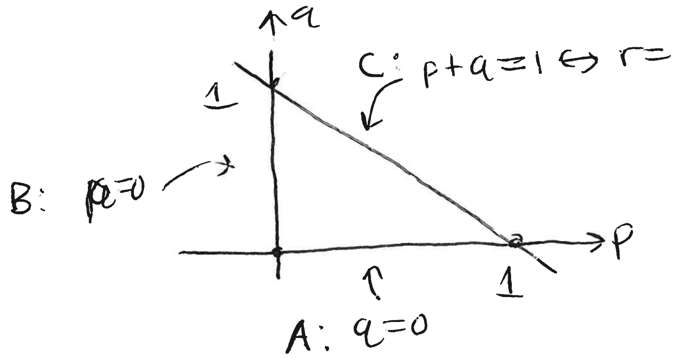
HW remarks 14.7.58

word problem. Yada Yada. $\longrightarrow$ Math extremization problem

Maximize $P = 2pq + 2pr + 2rq$ subject to constraints $\begin{cases} p>0, q>0, r>0 \\ p+q+r=1 \end{cases}$

solve constraint to eliminate one variable by expressing it in terms of the others: $r = 1-p-q > 0 \longrightarrow p+q<0$

so $P(p,q) = 2pq + 2(p+q)\underbrace{(1-p-q)}_{=r!}$



Maximize inside triangle or on boundary

A: $q=0$, $p=0..1 \rightarrow P = 2p(1-p)$

B: $p=0$, $q=0..1 \rightarrow P = 2q(1-q)$

C: $p+q=1 \rightarrow q=1-p$, $p=0..1 \rightarrow P = 2pq = 2p(1-p)$

} 1-d extremization on closed intervals

$P \sim$ downward parabola

max at midpoint between zeros

etc.

Interior: $P(p,q) \longrightarrow$ find crits, use second derivative test.

58. Three alleles (alternative versions of a gene) A, B, and O determine the four blood types A (AA or AO), B (BB or BO), O (OO), and AB. The Hardy-Weinberg Law states that the proportion of individuals in a population who carry two different alleles is

$$P = 2pq + 2pr + 2rq$$

where p, q, and r are the proportions of A, B, and O in the population. Use the fact that $p+q+r=1$ to show that P is at most $\frac{2}{3}$.