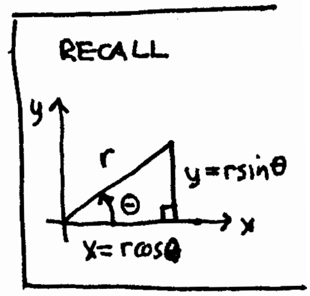
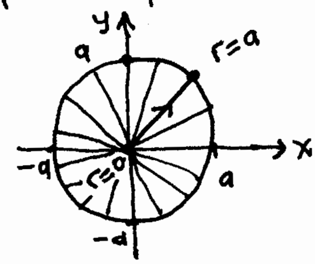
simple circles / cylinders / spheres in polar (r,θ) / cylindrical (r,θ,z) / spherical (ρ,θ,φ) coordinates

assume $a>0$

xy plane (circles in xy plane represent cylinders in xyz space)

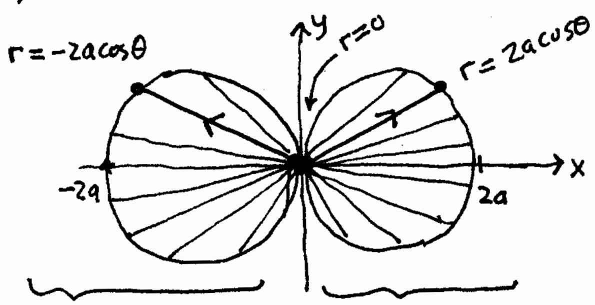


a) center at origin



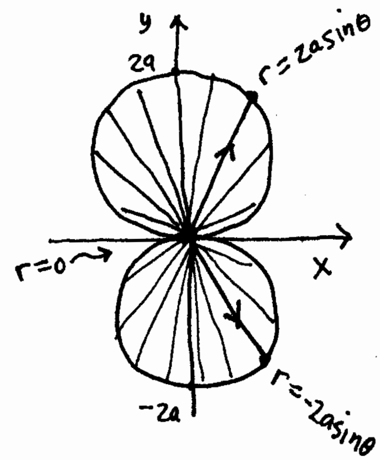
$x^2+y^2=a^2$
$r^2=a^2$
$r=a$
$\theta=0..2\pi$

b) center on axis, tangent to origin (θ interval between zeros of r)



$x^2+y^2=-2ax$
$r^2=-2ar\cos\theta$
$r=-2a\cos\theta$
$\theta=\frac{\pi}{2}..\frac{3\pi}{2}$

$x^2+y^2=2ax$
$r^2=2ar\cos\theta$
$r=2a\cos\theta$
$\theta=-\frac{\pi}{2}..\frac{\pi}{2}$

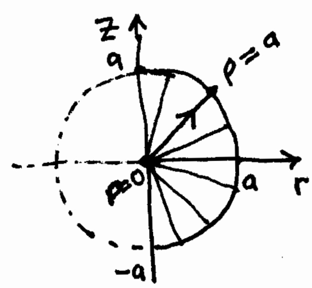


$x^2+y^2=2ay$
$r^2=2ar\sin\theta$
$r=2a\sin\theta$
$\theta=0..\pi$

$x^2+y^2=-2ay$
$r^2=-2ar\sin\theta$
$r=-2a\sin\theta$
$\theta=\pi..3\pi/2$
or
$\theta=-\pi..0$

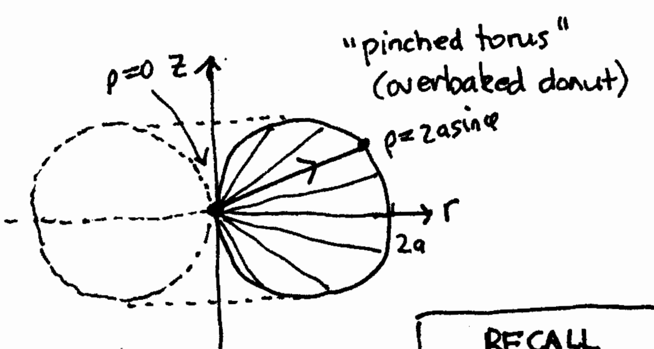
rz half plane $(x^2+y^2+z^2=r^2+z^2)$

a) sphere centered at origin

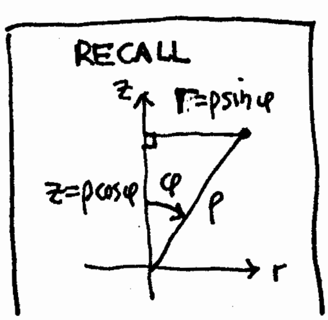


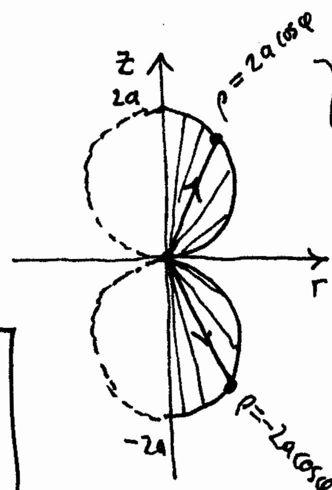
$r^2+z^2=a^2$
$\rho^2=a^2$
$\rho=a$
$\varphi=0..\pi$

b) sphere centered on axis, tangent to origin (φ interval between zeros of ρ)



$r^2+z^2=2ar$
$\rho^2=2a\rho\sin\varphi$
$\rho=2a\sin\varphi$
$\varphi=0..\pi$





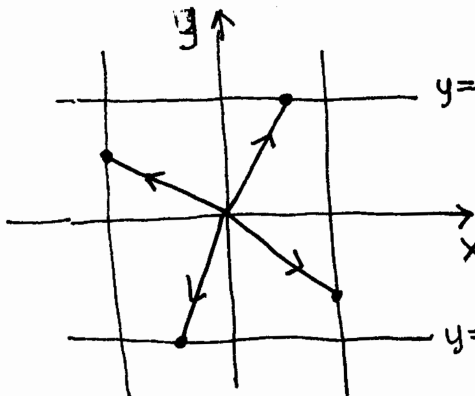
$r^2+z^2=2az$
$\rho^2=2a\rho\cos\varphi$
$\rho=2a\cos\varphi$
$\varphi=0..\pi/2$

$r^2+z^2=-2az$
$\rho^2=-2a\rho\cos\varphi$
$\rho=-2a\cos\varphi$
$\varphi=\frac{\pi}{2}..\pi$

simple lines/planes/cylinders in polar/cylindrical/spherical coordinates

assume $a > 0$

xy plane (lines in xy space are planes in xyz space)



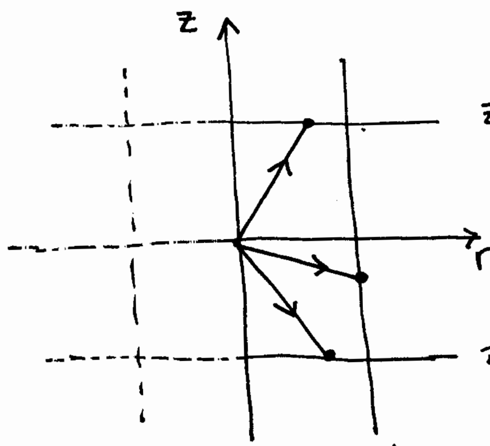
$y = a \to r\sin\theta = a$
$r = a\csc\theta$
$\theta = 0..\pi$

$y = -a \to r\sin\theta = -a$
$r = -a\csc\theta$
$\theta = \pi..2\pi$
or
$\theta = -\pi..0$

$x = -a \to r\cos\theta = -a$
$r = -a\sec\theta$
$\theta = \frac{\pi}{2}..\frac{3\pi}{2}$

$x = a \to r\cos\theta = a$
$r = a\sec\theta$
$\theta = -\frac{\pi}{2}..\frac{\pi}{2}$

rz half plane



$z = a \to \rho\cos\varphi = a$ (plane)
$\rho = a\sec\varphi$
$\varphi = 0..\pi/2$

$z = -a \to \rho\cos\varphi = -a$ (plane)
$\rho = -a\sec\varphi$
$\varphi = \frac{\pi}{2}..\pi$

$r = a \to \rho\sin\varphi = a$
$\rho = a\csc\varphi$
$\varphi = 0..\pi$
(cylinder)