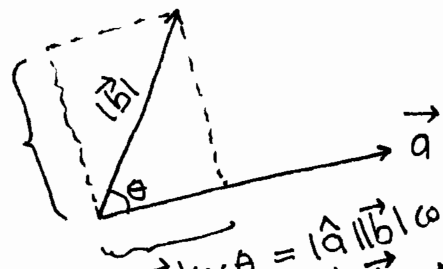
① Distance between a point and a line

Preliminary:

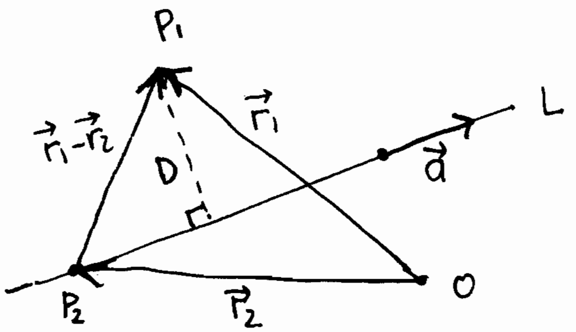
scalar component of $\vec{b}$ perpendicular to $\vec{a}$ is just $|\hat{a}\times\vec{b}|$.

$$|\vec{b}|\sin\theta = |\hat{a}||\vec{b}|\sin\theta = |\hat{a}\times\vec{b}|$$



$|\vec{b}|\cos\theta = |\hat{a}||\vec{b}|\cos\theta = \hat{a}\cdot\vec{b} \rightarrow$ magnitude: $|\hat{a}\cdot\vec{b}|$

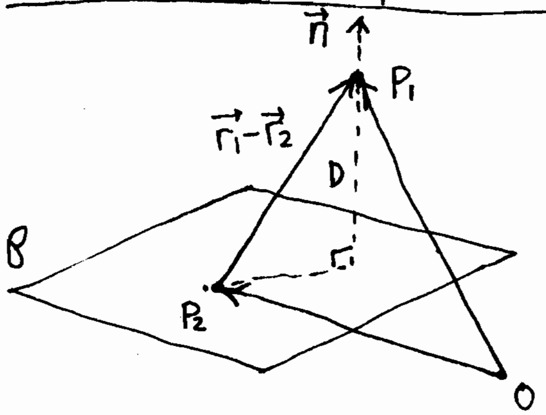
compare with scalar component of $\vec{b}$ along $\vec{a}$: ↗



Given a point P_1
find any point P_2 on line (see next page)
evaluate difference vector $\vec{r}_1-\vec{r}_2$
find its component perpendicular to the direction $\vec{a}$ of the line:

$$D = |\hat{a}\times(\vec{r}_1-\vec{r}_2)|$$

Distance between a point and a plane



Given a point P_1
find any point P_2 on the plane (next page)
evaluate the difference vector $\vec{r}_1-\vec{r}_2$
find its scalar component along the direction $\vec{n}$ normal to the plane
absolute value is the distance:

$$D = |\hat{n}\cdot(\vec{r}_1-\vec{r}_2)|$$

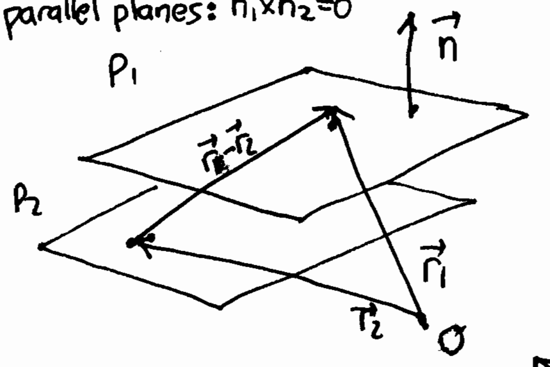
with these 2 projection operations we can extend these distance calculations easily to the distance between 2 parallel planes or between 2 skew lines (equivalent to 2 parallel planes) or between 2 parallel lines

(see next page)

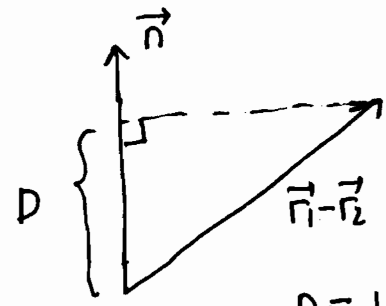
[we are not terribly interested in computing these distances. it only serves as practice with dot & cross products in projection operations]

② Distances between lines and planes

- 2 parallel planes: $\vec{n}_1 \times \vec{n}_2 = 0$



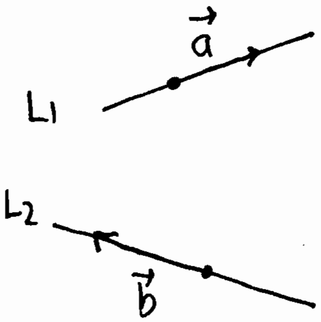
$\rightarrow$



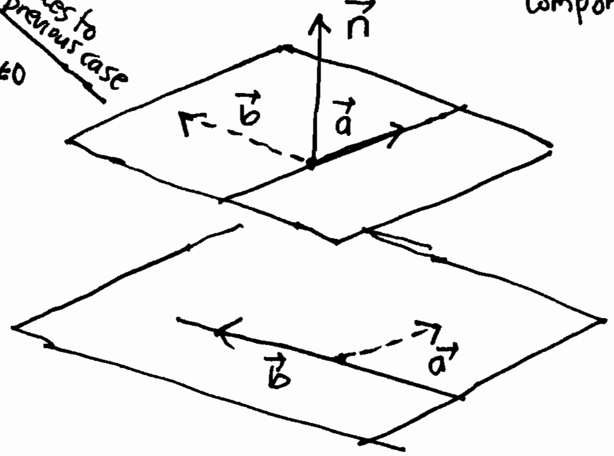
$$D = |\underbrace{\hat{n}\cdot(\vec{r}_1-\vec{r}_2)}_{\text{component along } \vec{n}}|$$

reduces to previous case

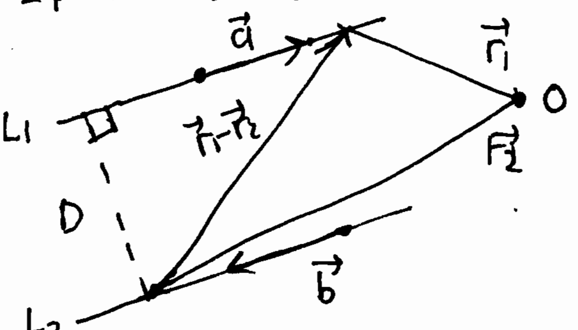
- 2 skew lines: $\vec{a}, \vec{b}$ not proportional: $\vec{a}\times\vec{b} \neq 0$



introduce $\xrightarrow{\quad}$ planes



- 2 parallel lines: $\vec{a}, \vec{b}$ proportional: $\vec{a}\times\vec{b} = 0$



$$D = |\underbrace{\hat{a}\times(\vec{r}_1-\vec{r}_2)}_{\text{component perp to } \vec{a} \text{ or } \vec{b}}|$$

One only needs to have a point on each plane or line to get a difference vector and then project that difference either along or perpendicular to the direction vector characterizing the plane or line to get a signed distance.

<u>point on a line:</u>

$$\left.\begin{aligned} x &= x_0 + at \\ y &= y_0 + bt \\ z &= z_0 + ct \end{aligned}\right\} \rightarrow t=0 \rightarrow \vec{r}_0 = \langle x_0, y_0, z_0\rangle$$

or

$$\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$$

<u>point on a plane:</u>

$$ax + by + cz = d$$

all planes intersect at least one axis

if $c \neq 0$ set $x = y = 0$ (z-axis) and solve for z.